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An Analytical Satellite Orbit Predictor (ASOP)

Mission Planning Analysis Division

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NASA

National Aeronautics and
Space Administration

Lyndon B. Johnson Space Center
Houston, Texas.

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SHUTTLE PROGRAM

An Analytical Satellite Orbit Predictor (ASOP)
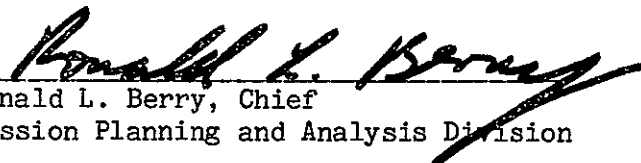
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National Aeronautics and Space Administration

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1.0 INTRODUCTION

This report contains the documentation and user's guide for the Analytical Satellite Orbit Predictor (ASOP) computer program. ASOP is based on mathematical methods that represent a new state-of-the-art for rapid orbit computation techniques. The theoretical development of these methods has been carried out during the past few years, and they are now in the form of an operational computer program. ASOP is intended to be used for computation of near-Earth orbits including those of the Shuttle/Orbiter and its payloads.

Orbit computation methods can usually be classified as:

a. Numerical methods - The calculations are carried out in a step-by-step manner. High precision is possible, but computer runtime can be exessive.

b. Analytical methods - The calculations are carried out in one step regardless of the prediction interval. Therefore, these methods have extremely fast computation times.

In the past, analytical methods have not been widely used because they were less accurate and required much more computer coding than numerical methods. The Poincaré-Similar elements (PSϕ) used in ASOP overcomes these disadvantages. It is possible to compute near-Earth orbits to within an accuracy of a few meters. Recursive equations are used instead of complicated formulas. Execution time of ASOP is on the order of a few milliseconds.

The theoretical foundation for the mathematical techniques used in ASOP were developed by Dr. Gerhard R. Scheifele (refs. 1 and 2). Scheifele developed the Delaunay-Similar elements (DSϕ) based on the true anomaly and used the elements to solve the J_2 perturbation problem (ref. 3). Later, Scheifele developed the Poincaré-Similar elements (PSϕ), which contain no singularities for zero eccentricity and zero inclination.

In reference 4, Mueller describes the relationship between the PSϕ elements and the Cartesian coordinates and establishes the PSϕ perturbed equations of motion. These elements and the associated equations of motion were used by Bond (ref. 5) to develop a nonsingular analytical solution to the J_2 perturbation problem. In 1977 the analytical solution was expanded by Scheifele (ref. 6) and Mueller (ref. 7) to include the perturbations due to atmospheric drag. Later, Mueller (ref. 8) developed the higher order zonal geopotential terms that have been implemented and documented by Wang (ref. 9). The development of time-dependent (tesseral) geopotential perturbation theory has been completed and is described by Mueller in reference 10. All of the current analytical theory has been implemented into ASOP including the zonal long period geopotential terms.

The ASOP program has been designed in two versions: (1) a stand-alone version that can be used interactively to obtain immediate results to a specific problem and (2) a user-subroutine package that can be incorporated into other software systems. Both versions were designed to be small and to execute quickly.

Both versions of the ASOP program have been written in UNIVAC standard FORTRAN-V and are available to the public on file NUMEG under the qualifier FM6-N08569 on the UNIVAC 1110-Exec 8 system. This document is intended to instruct the user in the operation of the ASOP program on this machine and to document the individual ASOP subroutines.

2.0 USER'S GUIDE

This section is intended to give the user all the information necessary to operate the ASOP programs. Because the program is designed to operate in two modes (stand-alone and subroutine package), each mode of operation is described separately.

The first part of this section (sec. 2.1) will describe the general input parameters and options available when using the stand-alone ASOP program. Also described in this section are the standard default values and the typical commands needed to execute the program in the demand or the batch mode. Finally some sample output is given to help the user if modifications are to be made to the system.

Section 2.2 will deal with the ASOP subroutine package. This section will describe the necessary modules that are used within the package as well as any interface requirement that the user must be aware of if he is to include this package in his own software. The input to and output from the ASOP subroutine are also fully described in this section, as are the subroutine's default values.

2.1 INDEPENDENT PROGRAM

The ASOP program was designed as an interactive program capable of giving the user fast, accurate answers to Shuttle-type orbit problems. The program may also be run in a batch environment if a large number of cases are to be investigated.

There are two basic methods used to control the operation of the ASOP program: flags and direct-user interaction. The flags are used to indicate the type of data being entered and to select certain options within the program. Direct interaction allows the user to check the input data to ensure their accuracy before continuing.

Primary data input to the ASOP program is accomplished using the NAMELIST '$INPUT'. The necessary input variables and user responses to program questions are described in section 2.1.1, and the program default values are described in section 2.1.2. Section 2.1.3 explains the printed output generated by the ASOP program. Finally, sections 2.1.4 and 2.1.5 describe the commands and user response required to run the ASOP program and give an example of the resulting output.

2.1.1 Input Description

The NAMELIST is the primary method of getting data into the ASOP program. However, during normal operation, the user is expected to interact with the program by supplying additional information. After starting the ASOP program (sec. 2.1.4), it will ask for the NAMELIST data with the statement

        INPUT DATA USING NAMELIST '$INPUT'

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At this point, the user has three options:

a. To enter the NAMELIST data directly from the keyboard.

b. To add a data file or element containing the NAMELIST information using the @ADD command (ref. 11).

c. To enter @EOF to terminate program execution.

If option C is selected, the program will respond with

NORMAL PROGRAM TERMINATION

and the program will stop.. If option A or B is selected the program will print out all the NAMELIST variables and their associated values (including default), as well as the initial conditions of the problem. The message

ENTER: X = EXECUTE; S = STOP; C = CHANGE INPUT

should then appear. Here, the user should check the input data and enter the necessary letter. If an X is entered, the program will continue the execution as directed by the input. When the input stop condition is satisfied, the program will again ask for data input as described earlier. The series of instructions can be repeated as often as necessary.

If an S is entered in response to the message, then the program will respond with **NORMAL PROGRAM TERMINATION** and program execution will terminate.

Should a C, or any other alphanumeric character, be entered at this point, then the message

CHANGE DATA USING THE NAMELIST '$INPUT'

will be displayed. The user can then reenter those values that are incorrect. Once the necessary corrections have been made, the program will again display the input data, the initial conditions, and the message

ENTER: X = EXECUTE: S = STOP: C = CHANGE INPUT

Table I describes the input variables that may be used in the NAMELIST '$INPUT'. Whether keying in the information or creating a data element, a ƀ$INPUT must be entered first where the ƀ represents one or more spaces. · Each variable entered must be preceded by one or more spaces, and if more than one variable is

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to appear on a line, they must be separated by a blank or a comma (,). To termi-
nate the NAMELIST input a Ƀ$END or Ƀ$ must be the last item entered.
See reference 12, pages 6 through 13, for a complete description of a NAMELIST
statement.

TABLE I.- NAMELIST INPUT VARIABLES

Variable	Type	No. of inputs required	Description and available options
EL	DP	6	<u>Must</u> be supplied by the user; may be given in the following forms as determined by the flag IEL EL (1) = X or a or h_a[b] EL (2) = Y or e or h_p[b] EL (3) = Z or i EL (4) = $\dot{X}$ or ω EL (5) = $\dot{Y}$ or Ω EL (6) = $\dot{Z}$ or M All angular input is given in degrees; all other values must be given in units as specified by the flag IUNITS
IEL	I	1	Flag determining the type of initial conditions input 1 = Keplerian elements[a] 2 = Cartesian coordinates 3 = Apogee and perigee[b]
STOP	DP	1	Final condition that must be satisfied in order to stop program execution normally
ISTOP	I	1	Flag that specifies the type of STOP condition 1 = STOP in days 2 = STOP in revolutions[a]
PRINT	DP	1	Increment for the printed output; a value is not needed if IPRINT is set to 0; PRINT = 0.0 is a valid entry

[a]Default value.
[b]To be implemented.

TABLE I.- Continued

Variable	Type	No. of inputs required	Description and available options
IPRINT	I	1	Flag that specifies the PRINT increment 0 = No PRINT increment[a] 1 = PRINT is days 2 = PRINT is revolutions
DATE	DP	6	Date of epoch given as a calendar date of the form Month, day, year, hours, minutes, seconds Note: Computation range is from March 1, 1900 through February 28, 2100
IDRAG	I	1	Flag that specifies if the drag equations are to be included in the computation 0 = No 1 = Yes[a]
CD	DP	1	Coefficient of drag, a value is not needed if IDRAG is set to 0
AREA	DP	1	Frontal surface area of the satellite; a value is not needed if IDRAG is set to 0
XMASS	DP	1	Total mass of the satellite in kilograms; a value is not needed if IDRAG is set to 0
ILONG	I	1	Flag that specifies the type of potential terms to be included in the computations 0 = None (two-body orbit) 1 = J_2 short period, and first-order secular terms[a] 2 = Compute the mean energy due to geopotential terms as defined by NMAX and MMAX

[a]Default value.

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TABLE I.- Concluded

Variable	Type	No. of inputs required	Description and available options
NMAX	I	1	Total number of zonal terms to be included by the geopotential model; a value is needed only if ILONG is set to 2
MMAX	I	1	Total number of tesseral terms to be included by the geopotential model; a value is needed only if ILONG is set to 2
IPSPRT	I	1	Flag to determine if the PS elements are to be included with all printout 0 = No[a] 1 = Yes
IUNITS	I	1	Flag that specifies the units of the input data and selects the appropriate physical constants 1 = km, sec[a] 2 = nm, sec 3 = ft, sec 4 = m, sec 5 = km, hr 6 = nm, hr 7 = E.r., min

[a]Default value.

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2.1.2 Default Values

To help shorten the number of data values that must be supplied by the user, the ASOP program assumes certain default values for those variables not explicitly mentioned on the input NAMELIST. These default values are listed in table II and a description of the variables can be found in section 2.1.1. Any variable not listed in table II must be specified by the user.

TABLE II.- DEFAULT NAMELIST VALUES

Variable	Default value
IEL	1
STOP	100.0
ISTOP	2
PRINT	0.0
IPRINT	0
DATE	1.,1.,1978.,0.,0.,0.0
IDRAG	1
AREA	185.3
CD	2.2
XMASS	90700.0
ILONG	1
NMAX	2
.MMAX	0
IPSPRT	0
IUNITS	1

2.1.3 Output Description

After the ASOP program has been started with the command

@XQT *NUMEG.ASOP-PROG

and the input data has been added, the program will print out all of the NAMELIST variables as shown in figure 1 and the initial conditions shown in figure 2.

```
$INPUT
EL      =      .3542070549999999990D+004.        .5256178579999999
99D+004.        .2152207510000000000D+004.       -.6414783099999999
99D+001,        .3115450500000000000D+001.        .2956928820000000
00D+001
IEL     =      +2
STOP    =      .1000000000000000000D+003
ISTOP   =      +2
PRINT   =      .5000000000000000000D+002
IPPINT  =      +2
DATE    =      .1000000000000000000D+001,        .1000000000000000
00D+001,        .1978000000000000000D+004.        .0000000000000000
00D+000,        .0000000000000000000D+000.        .0000000000000000
00D+000
IDRPG   =      +1
CD      =      .2200000000000000000D+001
APEA    =      .1853000000000000000D+003
CMASS   =      .9070000000000000000D+005
ILONG   =      +2
NMAX    =      +4
MMAX    =      +4
IPSPPT  =      +1
IUNITS  =      +1
$END
```

Figure 1.- NAMELIST data output format.

```
                          INITIAL CONDITIONS
  01/01/1978 00:00:   .000                          2443509.5000000
  DAYS: 0.0000000       ! REVS: 0.0000000    !Y!CHECK: 0.0000000
-----------------------+--------------------+----------------------
   A= 6.6993532+03 KM ! E=  .0009100       ! I= 30.00000    DEG
OMEGA= 17.92236    DEG !NODE= 20.00000  DEG ! M= 22.05838    DEG
   X= 3.5420705+03 KM !  Y= 5.2561786+03 KM ! Z= 2.1522075+03 KM
  VX=-6.4147831+00 KM/S!  VY= 3.1154505+00 KM/S!VZ= 2.9569288+00 KM/S
PS ELEMENTS:
   .10475457+001   .59495166-002  -.40245815+002   .59373237+000
   .51675614+005   .86357683-001   .11057447+003   .29769363+002
ENTER: X= EXECUTE; S= STOP; C= CHANGE INPUT
```

Figure 2.- Initial condition format.

After the initial conditions have been displayed, the program will wait for the user to check the input parameters. Some checks that can be made are

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a. CHECK value should be 0.0000000.

b. DAYS and REVS should be 0.0000000.

c. A double asterisk (**) will appear after the output condition that is being satisfied, i.e., after the DAYS or the REVS value. This will agree with input value of IPRINT or ISTOP.

d. Are the initial conditions (a, e, i, etc.) the desired values?

e. Are the units correct (km, km/s, etc.)?

Once the user is satisfied that the printed initial conditions are the ones needed, an X should be entered. When the X is entered, the program will continue execution and print out information as specified by the input parameters, i.e., STOP, ISTOP, PRINT, AND IPRINT (figure 3).

```
ENTER: X= EXECUTE; S= STOP; C= CHANGE INPUT
>X
 ASOP OUTPUT                                                PAGE: 0002
 01/04/1978 03:38:59.076                           2443512.6520726
 DAYS: 3.1520726+00      ! REVS: 5.0000000+01   **!CHECK:-8.7947667-06
-----------------------------+----------------------------+----------------
    A= 6.6965691+03 KM !     E=  .0011554        ! I=   29.98628      DEG
OMEGA=   31.71316    DEG !NODE= 356.98032    DEG ! M=   30.94208      DEG
   X= 3.3329544+03 KM !     Y= 4.9814775+03 KM  ! Z= 2.9717888+03 KM
  VX=-6.6907303+00 KM/S!    VY= 3.4262523+00 KM/S!VZ= 1.7709280+00 KM/S
PS ELEMENTS:
       .30741138+002 -.47544951-001  .61953616+001  .27234011+006
       .51684876+005  .18443874+000  .11744277+003  .29773531+002
---------------------------------------------------------------------------
                         FINAL CONDITIONS
 01/07/1978 07:17:  .596                           2443515.8034791
 DAYS: 6.3034791+00      ! REVS: 1.0000000+02   **!CHECK: 9.9538672-08
-----------------------------+----------------------------+----------------
    A= 6.6946148+03 KM !     E=  .0014630        ! I=   29.97883      DEG
OMEGA=   48.11705    DEG !NODE= 333.93645    DEG ! M=   37.22867      DEG
   X= 3.0136710+03 KM !     Y= 4.9535461+03 KM  ! Z= 3.3307289+03 KM
  VX=-6.6812825+00 KM/S!    VY= 3.8657270+00 KM/S!VZ= 3.0981351-01 KM/S
PS ELEMENTS:
       .83717851+002 -.76850190-001  .51655997+002  .54462213+006
       .51657340+005  .27972934+000  .10561303+003  .29777608+002
---------------------------------------------------------------------------
 INPUT DATA USING THE NAMELIST '$INPUT'
```

Figure 3.- Intermediate and final output format.

ASOP will always print the conditions as stated by PRINT and STOP unless it is unable to do so because of satellite reentry. The reentry condition is defined as the point where ASOP determines the satellite will reenter within one revolution. A sample of reentry is shown in figure 4.

```
                                 INITIAL CONDITIONS
 12/02/1978 00:00:   .000                            2443844.5000000
 DAYS: 0.0000000         !! ! REVS: 0.0000000      !CHECK: 0.0000000
----------------------------------+-------------------+------------------
    A= 6.5201400+03 KM  !   E=   .0000000          ! I=   28.60000   DEG
OMEGA=    .00000    DEG !NODE=  20.00000     DEG ! M=     .00000   DEG
    X= 6.1369274+03 KM  !   Y= 2.2300192+03 KM  ! Z= 0.0000000   KM
   VX=-2.3478950+00 KM/S!  VY= 6.4507885+00 KM/S!VZ= 3.7428027+00 KM/S
PS ELEMENTS:
     .34906585+000   .79599263-001  -.38148354+002  -.80044931-018
     .50979791+005  -.21869718+000   .10481174+003   .30593443+002
 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
ENTER: X= EXECUTE; S= STOP; C= CHANGE INPUT
:!!
 ASOP OUTPUT                                                PAGE: 0002
--------------------------------------------------------------------------
                                 REENTRY CONDITIONS
 12/02/1978 01:35:15.750                            2443844.5661545
 DAYS: 6.6154507-02       ! REVS: 1.0946538+00   **!CHECK: 2.9364986-06
----------------------------------+-------------------+------------------
    A= 6.5086414+03 KM  !   E=   .0007424          ·! I=   28.58721   DEG
OMEGA=  82.07266   DEG !NODE=  19.49643     DEG ! M= 312.57730   DEG
    X= 3.9663220+03 KM  !   Y= 4.8441563+03 KM  ! Z= 1.7670120+03 KM
   VX=-6.0513800+00 KM/S!  VY= 3.8489442+00 KM/S!VZ= 3.0831513+00 KM/S
PS ELEMENTS:
     .34293973+000  -.17842732-001  -.37192987+002   .57148404+004
     .50934787+005  -.13947939+000   .10505056+003   .30645447+002
 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
INPUT DATA USING THE NAMELIST '$INPUT'
```

Figure 4.- Reentry format.

In some cases, ASOP may determine that the satellite will reenter within
one revolution of the initial conditions. Therefore, ASOP would print out
the initial conditions as the reentry conditions.

In general, all output is clearly labeled, but some terms should be explained
further.

DAYS: Total number of days elapsed since the starting epoch.

REVS: Total number of revolutions completed.

CHECK: Value indicating the accuracy of the analytical theory; although
         this value is necessary as a check on the theory, it is not
         a sufficient check.

PS ELEMENTS: The Poincare-Similar elements listed as

$$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4$$

$$\rho_1 \quad \rho_2 \quad \rho_3 \quad \rho_4$$

Double asterisk (**): Indicates the stopping condition being satisfied;
                       this flag will move between the DAYS and REVS
                       value as needed.

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2.1.4 Run Setup (Control Cards)

The ASOP program is written in standard FORTRAN V and designed to run on the NASA/JSC UNIVAC 1110 computer using the EXEC-8 operating system. All the relocatable and executable elements are on the file FM6-NO8569*NUMEG. ASOP may be executed by entering the following for demand operation.

a. @QUAL FM6-NO8569

b. @ASG,A *NUMEG.

c. @XQT *NUMEG.ASOP-PROG

d. Add input data

e. Enter the letter X, S, or C (see section 2.1.3)

f. @EOF or go to step d.

g. @FIN

If run in a batch mode, the following input cards are needed.

a. @QUAL FM6-NO8569

b. @ASG,A *NUMEG.

c. @XQT *NUMEG.ASOP-PROG

d. Add data file or data cards

e. Card containing the letter X in column 1.

f. Repeat instructions d and e as often as necessary.

g. @EOF

h. @FIN

2.1.5 Sample Computer Run

In this section, a sample computer run is reproduced for a typical Shuttle-type orbit. The orbit has been predicted for 100 revolutions ($\approx$ 6.3 days) with the output given every three days.

This example is intended to familiarize the user with the format of the ASOP output and to illustrate the use of the various input options discussed in section 2.1.1. A full description of the output is given in section 2.1.3.

Initial parameters

Semimajor axis (a)	6699.3532 km (1.05 ER)
Eccentricity (e)	.001
Inclination (i)	30 degrees
Argument of perigee (ω)	18 degrees
Argument of the ascending node (Ω)	20 degrees
Mean anomaly (M)	22 degrees

Input parameters under the NAMELIST $INPUT

```
$INPUT
EL(1)    = 6599.3532
EL(2)    = .001
EL(3)    = 30.0
EL(4)    = 18.0
EL(5)    = 20.0
EL(6)    = 22.0
IEL      = 1
STOP     = 100.0
ISTOP    = 2
PRINT    = 3.0
IPRINT   = 1
DATE     = (Default Values Used)
IDRAG    = 1
CD       = 2.2
AREA     = 185.3
XMASS    = 90700.0
ILONG    = 2
NMAX     = 8
MMAX     = 8
IPSPRT   = 1
IUNITS   = 1
$END
```

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Sample computer run.

```
.WHOT .:NUMEG.ASOP-PROG
 INPUT DATA USING THE NAMELIST '#INPUT'
:WADD #NUMEG.DATA-ASOP/LONG2
; #END
 #INPUT
 EL        =              .6699353199999999998D+004,        .9999999999999999
970-003,
                     . .30000000000000000D+002.             .1800000000000000
000+002,
                       .20000000000000000D+002.             .2200000000000000
000-002
 IEL       =                  +1
 STOP      =              .10000000000000000D+003
 ISTOP     =                  +2
 PRINT     =              .30000000000000000D+001
 IPPINT    =                  +1
 DATE      =              .10000000000000000D+001,          .1000000000000000
000+001,
                         .19780000000000000D+004,           .0000000000000000
000+000,
                         .00000000000000000D+000,           .0000000000000000
000+000
 IDPAG     =                  +1
 CD        =              .22000000000000000D+001
 AREA      =              .18530000000000000D+003
 GMASS     =              .90700000000000000D+005
 ILONG     =                  +2
 NMAX      =                  +8
 MMAX      =                  +8
 IPSPRT    =                  +1
 IUNITS    =                  +1
 #END
----------------------------------------------------------------------------
                          INITIAL CONDITIONS
   01/01/1978 00:00:  .000                          2443509.5000000
   DAYS: 0.0000000       .: ! REVS: 0.0000000      'CHECK: 0.0000000
----------------------------+------------------------+-------------------
      A= 6.6993532+03 FM !   E=   .0010000      !  I=  30.00000     DEG
 OMEGA=  18.00000    DEG !NODE=  20.00000    DEG !  M=  22.00000    DEG
      .:= 3.5395374+03 FM !   X= 5.2568222+03 FM !  Z= 2.1530569+03 FM
     VX=-6.9168287+00 FM/S!  VY= 3.1134747+00 FM/S!VZ= 2.9562608+00 FM/S
 PS ELEMENTS:
         .10479476+001 -.76954051-002 -.40245812+002  .65091474+000
         .51675611+005  .10285596+000  .11057446+003  .29769235+002
 ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
 ENTER: .:= EXECUTE; S= STOP; C= CHANGE INPUT
 >
```

Sample computer run.- Concluded

```
ENTER: X= EXECUTE; S= STOP; C= CHANGE INPUT
.X
 ASOP OUTPUT                                              PAGE: 0002
 01/04/1978 00:00:  .000                          2443512.5000000
 DAYS: 3.0000000+00  *I ! REVS: 4.7586650+01   !CHECK: 1.4409285-05
----------------------------+----------------------+----------------------
   A= 6.6955919+03 KM !   E=  .0010075      ! I=  29.97867     DEG
OMEGA= 318.48280   DEG !NODE= 358.05518   DEG ! M= 314.47280    DEG
   X= 1.3674173+02 KM !   Y=-5.7964700+03 KM ! Z=-3.3390751+03 KM
  VX= 7.7162475+00 KM/S!  VY= 7.8156721-02 KM/S!VZ= 1.9615716-01 !M/S
PS ELEMENTS.
        .23578147+002  .10185805+000  .39900710+001  .25919875+006
        .51661116+005  .16527396+000  .11750491+003  .29773243+002
INITIALIZATION TIME =  569 MS.
 01/07/1978 00:00:  .000                          2443515.5000000
 DAYS: 6.0000000+00  ** ! REVS: 9.5183935+01   !CHECK: 1.1420590-05
----------------------------+----------------------+----------------------
   A= 6.6980353+03 KM !   E=  .0011075      ! I=  30.00575     DEG
OMEGA=  88.10042   DEG !NODE= 336.11077   DEG ! M=  61.22178    DEG
   X=-4.0856234+03 KM !   Y= 5.0245406+03 KM ! Z= 1.6975347+03 KM
  VX=-5.9157146+00 KM/S!  VY=-3.6769523+00 KM/S!VZ=-3.3249412+00 KM/S
PS ELEMENTS:
        .52458789+002 -.82058586-001  .47660244+002  .51840169+006
        .51671700+005  .21280088+000  .10760608+003  .29777083+002
----------------------------+----------------------+----------------------
                         FINAL CONDITIONS
 01/07/1978 07:17: 7.391                          2443515.8035578
 DAYS: 6.3035578+00       ! REVS: 1.0000000+02  ** !CHECK:-5.9225315-06
----------------------------+----------------------+----------------------
   A= 6.6946143+03 KM !   E=  .0014899      ! I=  29.97886     DEG
OMEGA=  51.13169   DEG !NODE= 333.93334   DEG ! M=  34.24363    DEG
   X= 3.0112883+03 KM !   Y= 4.9544626+03 KM ! Z= 3.3306205+03 KM
  VX=-6.6831533+00 KM/S!  VY= 3.8636606+00 KM/S!VZ= 3.0802620-01 !M/S
PS ELEMENTS:
       -.18745875+003 -.93856906-001  .51661785+002  .54462885+006
        .51657337+005  .27737512+000  .10561032+003  .29777855+002
INPUT DATA USING THE NAMELIST 'FINPUT'
*EOF
!! NORMAL PROGRAM TERMINATION !!
```

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In the interactive mode, the program solicits input data with the messages:

a. INPUT DATA USING THE NAMELIST '$INPUT'

b. ENTER: X = EXECUTE; S = STOP; C = CHANGE DATA

c. INPUT NEW DATA USING THE NAMELIST '$INPUT'

In the batch mode, the data input must be followed by one card containing the letter X as shown in the following examples:

<u>Example (1)</u>

Ƅ$INPUT

.

.

.

necessary data cards
(see section 2.1.1)

.

.

.

Ƅ$END
X
Ƅ$INPUT

.

.

.

etc.

.

.

.

@EOF
@FIN

<u>Example (2)</u>

@ADD filename.data element

.

.

.

changes to data element
(if any)

.

.

.

Ƅ$END
X
@ADD filename.data element

.

.

.

etc.

.

.

.

@EOF
@FIN

(Ƅ is a blank that must be included)

2.2 SUBROUTINE PACKAGE

Along with the ASOP stand-alone program, there is a subroutine package that may be included in the user's software. This package is in the form of a relocatable element and is located in FM6-NO8569*NUMEG.ASOP-SUB. Its operation is identical with any user-written subroutine.

This section will describe the information needed by the user to ensure proper insertion and operation of the ASOP package within the user's software.

2.2.1 Required Subroutines

The ASOP subroutine package consists of 27 subroutines. These are a driver subroutine (ASOP) that controls the basic logic of the package, 10 general subroutines that perform the functions necessary to the analytical theory and 17 subroutines to initialize the drag model and the computation of mean energy.

These subroutines are

ASOP	Driver subroutine
CONST	Planetary and mathematical constants
DRAG	Adjust the PS elements to account for drag perturbations
GEOPOT	Determine Earth's gravitational potential
LONGPP	Compute first-order zonal long periodic perturbations and second order zonal secular perturbations
POTEXP	Compute mean energy due to tesseral and sectorial geopotential harmonics
PREPD	Initialize the drag model
PSANS	PS analytical J_2 theory
PSTOX	Transformation subroutine: PS elements to Cartesian coordinates
TIMEPS	Time iteration stopping procedure
XTOPS	Transformation subroutine: Cartesian coordinates to PS elements

Initializing subroutine called by CONST:

PREPT   Initialize the geopotential coefficients for the Earth

Computational subroutines called by LONGPP

DETERM	Compute the first-order long-period generating function and its derivatives
FPRIME	Compute the second-order zonal Hamiltonian and its derivatives

Initializing subroutines called by POTEXP:

COEFF	Compute the binomial coefficients and the Fourier coefficients of the powers of cosine and sine
ILOG10[a]	Determine the number of terms to be included in the expansion for the Earth's geopotential model
RECUR	Compute the sine and cosine of multiples of an angle
TIMEXP	Compute coefficients of the expansion of the time equation

---

[a]ILOG10 is also called by DETERM and FPRIME.

Initializing subroutines called by PREP:

CANFOR   Compute the canonical forces due to atmospheric drag for the PS equation

DENSTY   Compute the atmospheric density at a given altitude above an oblate Earth

INITAL   Initialize the coefficients for the Jacchia 71/Lineberry atmospheric density model

MATIN   Invert an n x n matrix and/or solve $Ax = B$

MTOECC   Convert the mean anomaly to eccentric anomaly and compute its sine and cosine

PREPS.   Establish the parameters needed to calculate the position of the Sun

SACT   Determine the solar activity coefficients for a given date

SUN   Compute the position of the Sun analytically

TABLE   Generate the table of coefficients for the sine and cosine of $n\varnothing_1$ for the atmospheric drag function

The subroutines listed above are fully described in section 3.3, and a diagram of the data flow between these subroutines can be found in appendix H.

To help the user add these subroutines to his own software, a relocatable element has been formed that includes all the above subroutines. Therefore, the user needs only to include the element

FM6-NO8569*NUMEG.ASOP-SUB

when forming an executable element.

2.2.2 Interface Requirements

To access the ASOP subroutine package, the programer must use the FORTRAN statement

CALL ASOP (X,STOP,ISTOP,NEWX)

A full description of the argument list variables can be found in sections 2.2.3 and 3.3.3. Also, the user must initialize certain COMMON block variables before entering the ASOP subroutine.

Table III gives a list of the variables that must be initialized prior to calling the ASOP subroutine and the COMMON block in which the variables are located. A complete description of the variables and their allowed values can be found in section 2.1.1.

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TABLE III.- COMMON BLOCK INITIALIZATION

Variable	COMMON block	Default value(s)
IUNITS	CPRINT	1
IDRAG	PERTRB	1
CD	DRAG	2.2
AREA	DRAG	185.3
XMASS	DRAG	90,700.0
DATE	EPOCH	1.0,1.0,1978.0, 0.0,0.0,0.0
XJDATE	EPOCH	2,443,509.5
ILONG	PERTRB	1
NMAX	TESS	2
MMAX	TESS	0

2.2.3 Input/Output Description

The argument list to the ASOP subroutine consists of four arguments given in the following order.

    CALL ASOP (X,STOP,ISTOP,NEWX)

On input the variables are

X   An array of eight elements corresponding to the initial state vector in the following order:

        X(1): X position component
        X(2): Y position component
        X(3): Z position component
        X(4): X velocity component
        X(5): Y velocity component
        X(6): Z velocity component
        X(7): Physical time (set to zero)
        X(8): Total energy (set to zero)

STOP   Stop value desired; it may be given in days or revolutions.

> **WARNING:** A number must not be used in the following positions of the argument list. Assign the desired value to a variable, and use the variable in the argument list. If a number is used in these positions instead of a variable, unpredictable results may occur.

ISTOP   Flag determining whether the value given to STOP is in days (ISTOP = 1) or revolutions (ISTOP = 2).

> **WARNING:** Only a 1 or 2 should be used as input. On output, ISTOP should be checked to see if ISTOP was reset to 3 indicating reentry conditions.

NEWX   Flag determining if the ASOP subroutine is to be initialized

NO = 0, YES = 1

The initialization process must be done whenever new initial conditions are entered.

Input to the ASOP program is also done by means of COMMON blocks. These COMMON block input variables control the internal operation of the ASOP subroutine package and should not be changed once the subroutine has been initialized.

Table III (section 2.2.2) gives a complete list of the variables that must be initialized and their default values.

On output the variables are

X   An array of eight elements corresponding to the final state vector at the given value of STOP. The order is the same as for input. If the value of STOP was given in days (ISTOP = 1), then the value of X(7) will be an approximate value of STOP given in the time units specified by IUNITS.

STOP   Unchanged from the input value.

ISTOP   Set to three (3) if reentry condition exists. Otherwise, it is unchanged from the input value. Therefore, a variable name should always occupy this position in the argument list.

NEWX   Set to zero (0) upon return from the ASOP subroutine; therefore, a variable name should always occupy this position in the argument list. Otherwise, unpredictable results may occur.

3.0 DESCRIPTION AND STRUCTURE OF ASOP

The ASOP program was designed as a top-down structured program consisting of 33 subroutines (modules) and a main (driver) program. Within this set of subroutines, a subset of 27 subroutines comprises the removable ASOP subroutine package. The package has been designed for easy incorporation into existing user software.

The two sets of subroutines are described in the following subsections and each subroutine is documented in section 3.3.

3.1 THE ASOP PROGRAM

Basically, the ASOP program, shown in figure 5, consists of four segments: a main program or driver, an input routine, an output routine, and a removable ASOP subroutine package. The removable package is described in section 3.2 so that only the first three segments will be described here.

The purpose of the main program is to call all the necessary subroutines (input, output, constants, etc.) in a specific sequence to produce the desired results. In particular, the main program provides for the repetitive call to the ASOP subroutine in order to produce a satellite ephemeris.

All input to the program is controlled by the subroutine INPUT. Its primary functions are

a. Set all default values

b. Accept input from the NAMELIST $INPUT

c. Convert any input values that require conversion

d. Issue normal program termination command

Output from the ASOP Program is performed only by the subroutine OUTPUT. This subroutine contains all the FORMAT specifications used for normal[a] output from ASOP. The output program will also perform any conversions required to make the output more understandable. This involves converting Cartesian coordinates to Keplerian elements, radians to degrees, and time units to days.

---

[a]All error output is controlled by the individual subroutines.
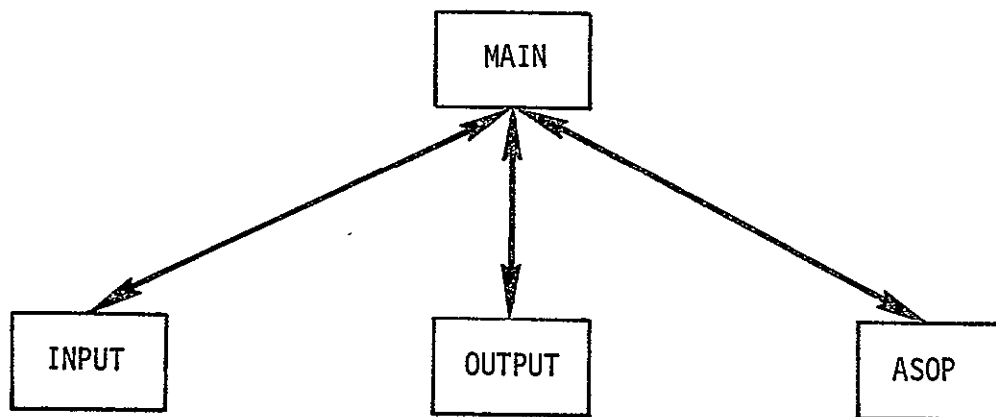
Figure 5.- Structure of the ASOP program.

3.2 THE ASOP SUBROUTINE PACKAGE

The ASOP subroutine package has been designed as an independent segment of the ASOP program so that it can be incorporated into existing software with little or no modifications.

This package has seven basic parts (fig. 6):

a. A driver subroutine (ASOP)

b. Coordinate transformations (XTOPS, PSTOX)

c. A stopping routine (TIMEPS)

d. The analytical theory (PSANS, LONGPP, DRAG)

e. Initialization of the geopotential model

f. Initialization of the drag model (PREPD)

g. Initialization of physical and trigonometric constants (CONST, PREPT)

Along with the 11 subroutines mentioned above there are an additional 16 subroutines. Subroutine GEOPOT performs computations required by the coordinate transformation subroutines XTOPS and PSTOX. Subroutines LONGPP, POTEXP, and PREPD use the remaining 15 subroutines for initialization of their respective perturbation models.

Subroutine ASOP performs the same task as the main program in that it calls all the necessary subroutines as dictated by the input values. In the case of the ASOP subroutine, however, the input values are given through an argument list and a few COMMON blocks. Therefore, if a satellite ephemeris is desired, the user must supply the necessary coding within his own software.

The basic input to the subroutine is:

a. The Cartesian coordinates (X)

b. A stop value (STOP)

c. A stop flag (ISTOP)

d. A new data flag (NEWX)

This input is fully described in sections 2.2.3 and 3.3.3.

Because the analytical theory has been developed in PS elements, it is necessary to perform transformations to and from the element set. Transformation into the PS elements from Cartesian coordinates is performed by the subroutine XTOPS while the reverse transformation is performed by the subroutine PSTOX.

A time-stop subroutine has been included because the PS elements use the true anomaly as the independent variable and do not use time. This stopping routine is an iterative procedure and is described in section 4.3.

Subroutine PSANS updates the PS elements at a specified value of the independent variable. The J_2 perturbations are computed within PSANS, the drag perturbations are computed by a call to subroutine DRAG, and the long-period geopotential terms are computed by LONGPP.
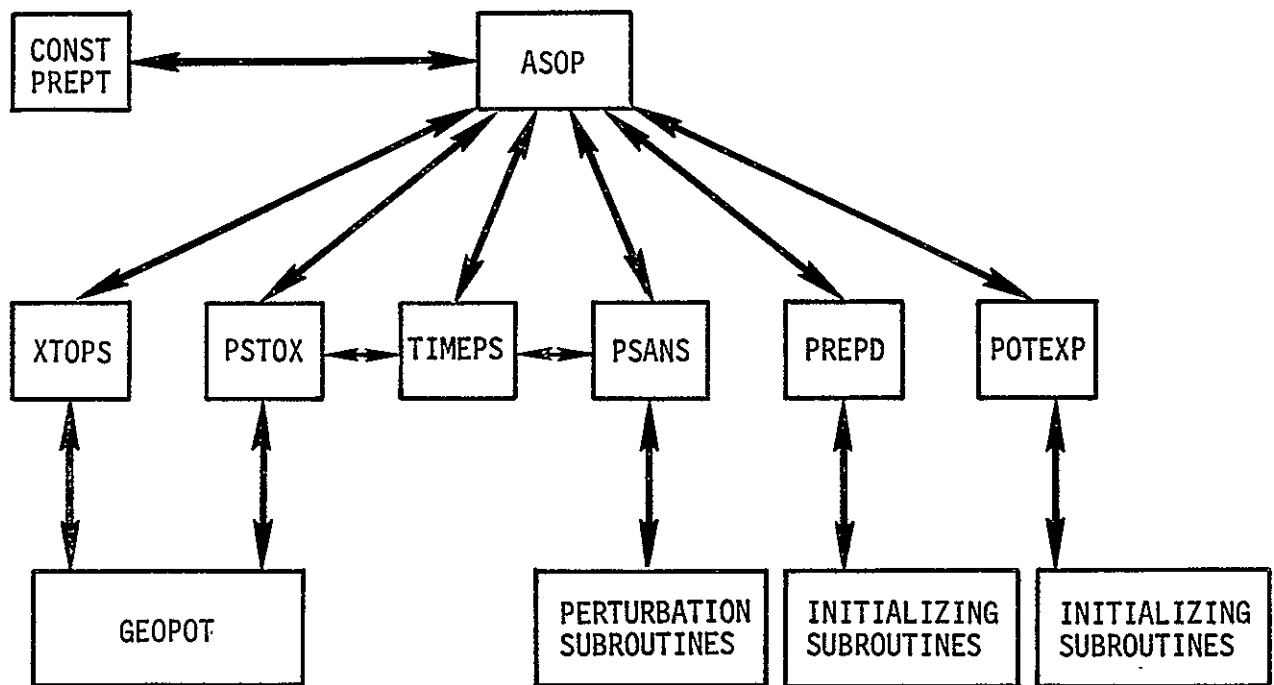
Figure 6.- Structure of the ASOP subroutines.

3.3 MODULE DESCRIPTIONS

This section will give a complete description of the subroutines currently used in the ASOP program. Each description will contain a brief statement as to the purpose, or use, of the subroutine as well as a description of important variables used within the subroutine. Also included are lists of the named COMMON blocks used, external references to other ASOP subroutines, and other ASOP subroutines that reference the subroutine being described. Information is also available as to the calling sequence of the subroutine and the size of the subroutine. Each description is followed by a general flow chart of the subroutine (figs. 7 through 40).

Each program is listed alphabetically, with the exception of the MAIN program, which is described first.

3.3.1 MAIN Program (Driver)

Purpose: Driver for the Analytical Satellite Orbit Predictor (ASOP) program

Calling sequence: None

Called by: Operating system

Subroutines/functions called: ASOP, INPUT, OUTPUT

Named COMMON:
/CARTC /	X(8),R,RI	
/CBASIC/	PI,TWOPI,DEG,RAD,DAY,DTOKM	
/CPRINT/	PRINT,IPRINT,IPSPRT,IUNITS	
/END /	STOP,ISTOP	
/PS /	SIG(8),TAU,TAUMAX,TAUINT	
/PSTIME/	CLO,FAKTS,TOL	

Equivalence: (X(7),TIME)

Program data: Size = 147_8 (103_{10}) words compiled.

FORTRAN variable	Dimension	Type	Input/ output	Description
DAY	1	DP	I	Conversion of days into hours, minutes, or seconds. Its numeric value is controlled by IUNITS
IPRINT	1	I	I/O	Flag to determine if the intermediate printout is to be given on days or revolutions. If reentry condition exists, flag is reset to 3.

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FORTRAN variable	Dimension	Type	Input/ output	Description
				= 0 no printout = 1 days = 2 revolutions = 3 reentry condition exists (output only)
ISTOP	1	I	I/O	Flag to determine if the program is to terminate after a given number of days or revolutions as specified by STOP. If reentry condition exists flag is reset to 3 = 1 days = 2 revolutions = 3 reentry condition exists (output only)
NEWX	1	I	I/O	Flag to determine if the ASOP program is to be initialized = 0 no = 1 yes
PRINT	1	DP	I	Increment for which the intermediate printout is desired (valid only if IPRINT $\geq$ 1)
STOP	1	DP	I	Value at which the program is to stop execution (units are determined by ISTOP)
STOPDT	1	DP	O	Value at which the next intermediate printout is desired (valid only if IPRINT $\geq$ 1 and PRINT > 0.0)
TAU	1	DP	I	Independent variable of the PS elements; it is defined such that REVS = TAU/2π
TAUINT	1	DP	I	Initial value of TAU for which the force model is valid

FORTRAN variable	Dimension	Type	Input/ output	Description
TOL	1	DP	I	Tolerance used by the time-stopping routine.
TWOPI	1	DP	I	2π
X	8	DP	I	Space vehicle parameters in the form: $X(1) \rightarrow X(3) = \vec{X}$ $X(4) \rightarrow X(6) = \vec{V}$ $X(7) = \text{time}$ $X(8) = \text{total energy}$
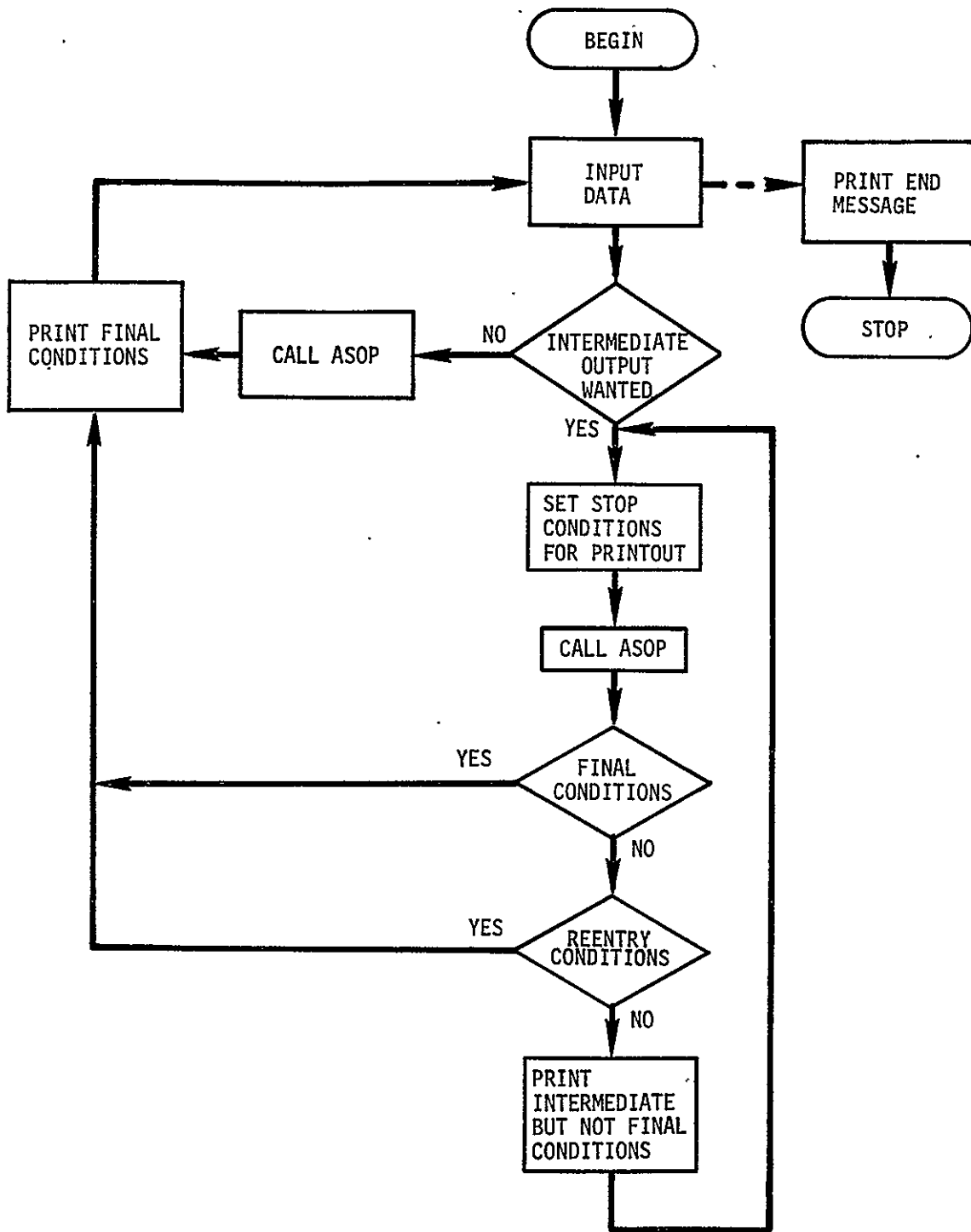
Final 7.- MAIN program flow charts.

3.3.2 AEIXYZ (Subroutine)

Purpose: Transform the Keplerian elements (a,e,i,ω,Ω,M) into Cartesian coordinates $(\vec{X},\vec{V})$

Calling sequence: CALL AEIXYZ

Called by: INPUT

Subroutines/functions used: MTOECC

Named COMMON: /CARTC / X(3),V(3),TIME,ENERGY,R,RI
       /CBODY / XMU,XMUI,SQTMU,SQTMUI,EPS
       /KEPLER/ A,E,XI,OMEGA,XNODE,XM
       /RPOOL / · SINC,SOMEGA,SNODE,CINC,COMEGA,CNODE,
            B1(3),B2(3),X11,X12,V11,V12,TEMP0,TEMP1

Equivalence: (XI,XX0(1)),(SINC,XX1(1)),(CINC,XX2(1))

Program data: Size = 302_8 (194_{10}) words compiled
      Subroutine valid only when $|e| < 1.0$

FORTRAN variable	Dimension	Type	Input/ output	Description
A	1	DP	I	Semimajor axis of the satellite's orbit; must be greater than 0
COSEA	1	DP	I	Cosine of the eccentric anomaly (cos E)
E	1	DP	I	Orbital eccentricity (e); must not be greater than 1
EA	1	DP	I	The eccentric anomaly of the satellite computed from Kepler's equation (rad)
OMEGA	1	DP	I	Argument of pericenter (ω) (rad)
R	1	DP	O	Magnitude of the position vector of the satellite
RI	1	DP	O	Inverse of R
SINEA	1	DP	I	Sine of the eccentric anomaly (sin E)

FORTRAN variable	Dimension	Type	Input/ output	Description
V	3	DP	O	Velocity vector of the satellite with respect to the Earth's equator $$V(1) = V_x$$ $$V(2) = V_y$$ $$V(3) = V_z$$
X	3	DP	O	Position vector of the satellite with respect to the Earth's equator $$X(1) = X$$ $$X(2) = Y$$ $$X(3) = Z$$
XI	1	DP	I	Orbital inclination to the Earth's equator (i,rad)
XM	1	DP	I	Mean anomaly of the satellite (M,rad)
XMU	1	DP	I	Gravitational constant for the central body (μ)
XNODE	1	DP	I	Argument of the ascending node (Ω,rad)
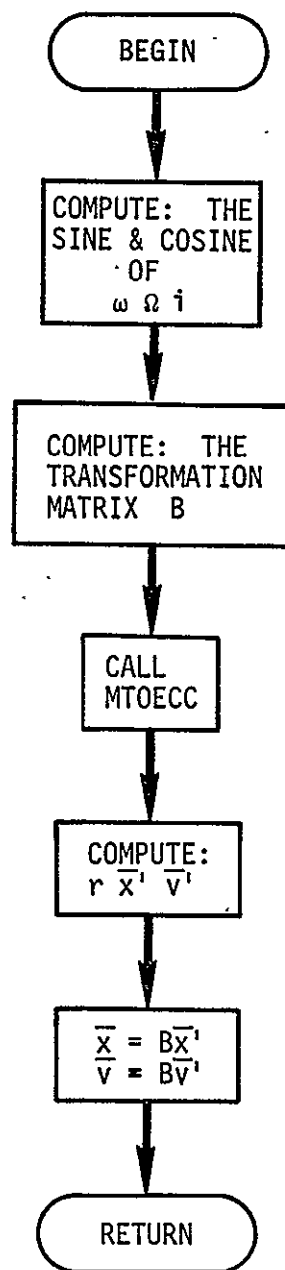
Figure 8.-AEIXYZ flow chart.

3.3.3 ASOP (Subroutine/Driver)

Purpose: Driver for the analytical section of the ASOP program; it controls all the operations required by the analytical program PSANS

Calling sequence: ·CALL ASOP (X,STOP,ISTOP,NEWX)

Called by: MAIN

Subroutines/functions used: CONST[a], POTEXP, PREPD, PSANS, PSTOX, TIMEPS, XTOPS

Named COMMON: /CARTC / XIN(8),R,RI
/CBASIC/ PI,TWOPI,DEG,RAD,DAY,DTOKM
/GEO / RE
/PERTRB/ IDRAG,ILONG
/PS / SIG(8),TAU,TAUMAX,TAUINT

Program data: Size = 256_8 (174_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
DAY	1	DP	I	Conversion of days into hours, minutes, or seconds
DTOKM	1	DP	I	Converts distance into kilometers
IDRAG	1	I	I	Flag to determine if the drag calculations are to be included
				= 0 no
				= 1 yes
ILONG	1	I	I	Flag to determine the type of geopotential terms to be included
				= 0 none (two-body orbit)
				= 1 J_2 short period, and first-order secular terms
				= 2 Compute the mean energy due to geopotential terms as defined by NMAX and MMAX (see table I for a description of NMAX and MMAX)

---

[a]Called only in subroutine package.

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FORTRAN variable	Dimension	Type	Input/ output	Description
ISET	1	I	I	Flag to determine if the force model must be updated = 0 no (Force model still valid) = 1 yes (Update state vector to τ_{max} and reinitialize force model)
ISTOP	1	DP	I/O	Flag to determine if the value of STOP is given as days or revolutions; if reentry conditions exist, flag is reset to 3 = 1 days = 2 revolutions = 3 reentry
NEWX	1	I	I/O	Flag to determine if the ASOP program is to be initialized = 0 no = 1 yes
R	1	DP	I	Magnitude of position vector
RE	1	DP	I	Equatorial radius of the central body (Earth)
RI	1	DP	I	Inverse magnitude of the position vector
STOP	1	DP	I	Value at which the final state vector is required; units are set by ISTOP
TAU	1	DP	O	Independent variable of the PS elements
TAUINT	1	DP	I/O	Initial value of τ for which the force model is valid; initially set to 0
TAUMAX	1	DP	I/O	Range of validity for the force model; when τ exceeds this value (τ_{max}), the force model must be reinitialized
TWOPI	1	DP	I	2π

FORTRAN variable	Dimension	Type	Input/ output	Description
X	8	DP	I/O	Initial/final state vector
				$X(1) \rightarrow X(3) = \vec{X}$ $X(4) \rightarrow X(6) = \vec{V}$ $X(7)$ = time $X(8)$ = total energy
				If initializing (NEWX = 1), then $X(7)$ and $X(8)$ will be set to 0
XIN	8	DP	I/O	Identical to X but allows the ASOP subroutine to be removed from the stand alone program
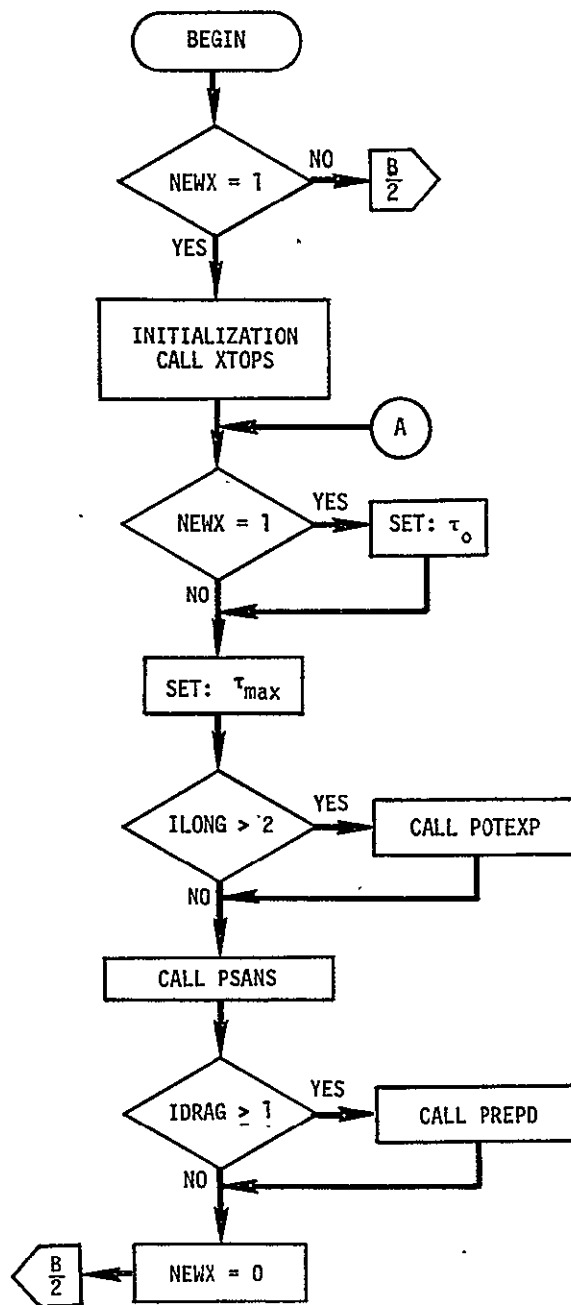
Figure 9.- ASOP flow chart.

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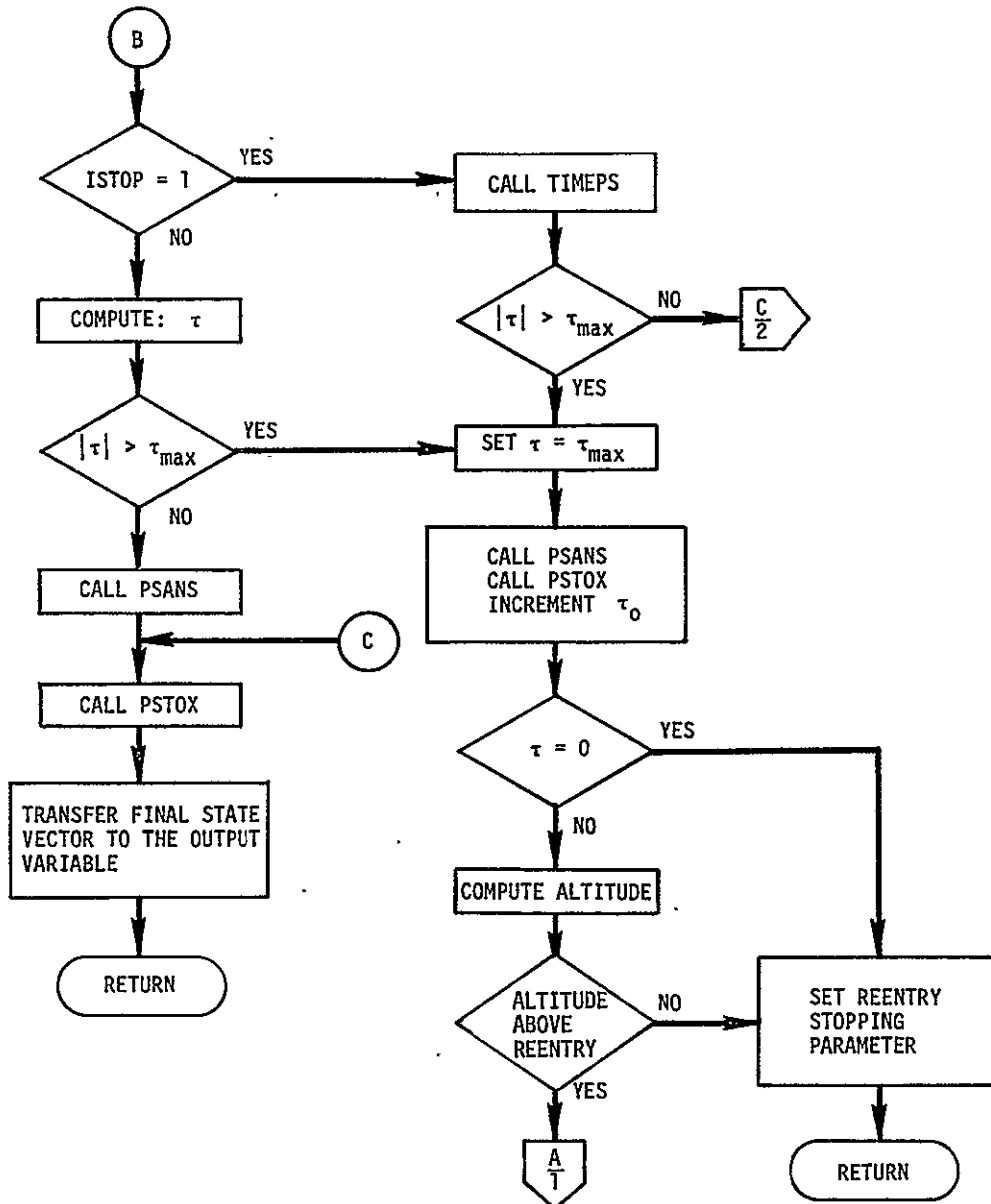
Figure 9.- Concluded.

3.3.4 CANFOR (Subroutine)

Purpose: Compute the canonical forces due to atmospheric drag for the PS equation used with the ASOP program (refs. 6 and 7)

Calling sequence: CALL CANFOR (CFORCE)

Called by: PREPD

Subroutines/functions used: None

Named COMMON:
/CBODY /	XMU,XMUI,SQTMU,SQTMUI,EPS
/DBETAS/	B,BSQ,B3,B4
/DTABLE/	T(12)
/GMTROT/	WE,THETAO
/PS /	S1,S2,S3,S4,R1,R2,R3,XL,TAU
/PSANS1/	SSIG1(2),TWOL,XIQL,FAK
/PSANS2/	SUM2,SUM3,DIFF2,DIFF3,GC,HC,PSSQRT,PS,QS
/RETRO /	IRO

Program data: Size = 524_8 (340_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
B	1	DP	I	β
BSQ	1	DP	I	β^2
CFORCE	8	DP	O	Drag force defined in PS elements
DIFF3	1	DP	I	$\rho_3^2 - \sigma_3^2$
GC	1	DP	I	G
HC	1	DP	I	H
IRO	1	I	I	Flag to determine if retrograde orbit
				$= -1$ yes
				$= 1$ no
PS	1	DP	I	p
R2	1	DP	I	ρ_2
R3	1	DP	I	ρ_3

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ORTRAN ariable	Dimension	Type	Input/ output	Description
UM3	1	DP	I	$\left(\sigma_3^2 + \rho_3^2\right)\big/2$
2	1	DP	I	σ_2
3	1	DP	I	σ_3
	12	DP	I	Table of averaged Fourier series in σ_1
WOL	1	DP	I	2L
E	1	DP	I	Rotational rate of the Earth
L	1	DP	I	ρ_4
MU	1	DP	I	μ
MUI	1	DP	I	$1/\mu$
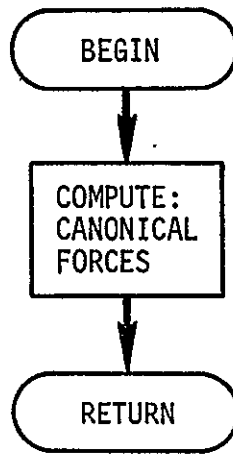
Figure 10.- CANFOR flow chart.

3.3.5 CDTOJD (Subroutine)

Purpose: Compute a Julian date corresponding to a given calendar date

Calling sequence: CALL CDTOJD (CDATE,XJDATE)

Called by: INPUT

Subroutines/functions used: None

Named COMMON: None

Program data: Size = 176_8 (126_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
CDATE	6	DP	I	A calendar date given in the form
				CDATE(1) = month
				CDATE(2) = day
				CDATE(3) = year
				CDATE(4) = hours
				CDATE(5) = minutes
				CDATE(6) = seconds
				Computation range is from March 1, 1900 through February 28, 2100
XJDATE	1	DP	O	Julian date corresponding to the given calendar date CDATE
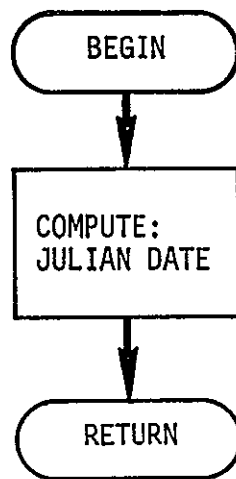
Figure 11.- CDTOJD flow chart.

3.3.6 COEFF (Subroutine)

Purpose: Compute the binomial coefficients A( ) and the Fourier coefficients
B( ) of the powers of cosine and sine

Calling sequence: CALL COEFF

Called by: POTEXP

Subroutines/functions used: None

Named COMMON: /EXPCOF/   A(200),B(200),NDEX0,NDEX(18),IEXPFL

Program data: Size = 322_8 (210_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
A	200	DP	O	Array containing binomial coefficients
B	200	DP	O	Array containing Fourier coefficients for cosine and sine raised to a power
IEXPFL	1	I	O	Flag to determine if the A, B arrays have been computed = 0 no = 1 yes
NDEX	18	I	O	Array of pointers to A and B coefficients
NDEX0	1	I	O	Zero Index of table NDEX

```
                    ┌─────────┐
                    │  BEGIN  │
                    └─────────┘
                         │
                         ▼
          ┌──────────────────────────────┐
          │                              │
          │  COMPUTE:                    │
          │  BINOMIAL                    │
          │  AND FOURIER                 │
          │  COEFFICIENTS                │
          │  OF THE POWERS               │
          │  OF SINE AND COSINE          │
          │                              │
          └──────────────────────────────┘
                         │
                         ▼
                    ┌─────────┐
                    │ RETURN  │
                    └─────────┘
```
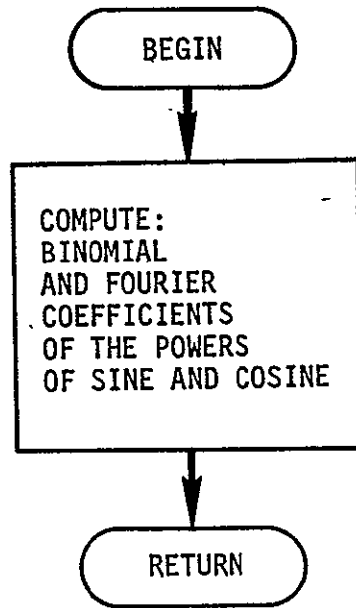
Figure 12.- COEFF flow chart.

3.3.7 CONST (Subroutine)

Purpose: Initialize the mathematical and physical constants needed to execute the ASOP program (refs. 13 and 14).

Calling sequence: CALL CONST

Called by: ASOP[a], INPUT

Subroutines/functions used: PREPS,PREPT

Named COMMON: /CBASIC/   PI,TWOPI,DEG,RAD,DAY,DTOKM
                /CBODY /   XMU,XMUI,SQTMU,SQTMUI,EPS
                /CPRINT/   X,I(2),IUNITS
                /DRAG /   CD,AREA,XMASS,CDRAG
                /END   /   STOP,ISTOP
                /EPOCH /   CDATE(6),XJDATE
                /GEO   /   RE,CJ2,CS(187),SS(187),IGEOFL
                /GMTROT/   WE,THETAO
                /PERTRB/   IDRAG,ILONG

Equivalence: (DT2,DT1),(THETAO,GMST),(XJDATE,XJED)

Program data: Size = 406_8 (262_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
AREA	1	DP	I	Cross sectional surface area of the satellite (m^2) (needed only if IDRAG $\geq$ 1)
CD	1	DP	I	C_d coefficient of drag (needed only if IDRAG $\geq$ 1)
CDRAG	1	DP	O	Drag coefficient $= C_d \dfrac{A}{2m}$ (m^2/kg)
CJ2	1	DP	O	J_2 coefficient of the central body
DAY	1	DP	O	Value to convert days into seconds, minutes, or hours

[a]Called only by subroutine package.

FORTRAN variable	Dimension	Type	Input/ output	Description
DEG	1	DP	O	$180/\pi$
DTOKM	1.	DP	O	Converts distance into kilometers
EPS	1	DP	O	$3/2 \ (\mu \ J_2 \ R_e^2)$
GMST	1	DP	I	Initial hour angle of Earth
IDRAG	1	I	I	Flag to determine if the drag calculations are to be included = 0 no = 1 yes
ILONG	1	I	I	Flag to determine the type of geopotential terms to be used = 0 none (two-body orbit) = 1 J_2 short period, and first-order secular terms = 2 Compute the mean energy due to geopotential terms as defined by NMAX and MMAX
IUNITS	1	I	I	Flag to determine what units are to be used for the calculations = 1 km,sec = 2 nm,sec = 3 ft,sec = 4 m,sec = 5 km,hr = 6 nm,hr = 7 E.r.,min
PI	1	DP	O	π
RAD	1	DP	O	$\pi/180$
RE	1	DP	O	Equatorial radius of central body (Earth)
SQTMU	1	DP	O	$\sqrt{\mu}$
SQTMUI	1	DP	O	$1/\sqrt{\mu}$
TWOPI	1	DP	O	2π

FORTRAN variable	Dimension	Type	Input/ output	Description
WE	1	DP	O	Rotational rate of Earth
XJED	1	DP	I	Julian date
XMASS	1	DP	I	Total mass of the satellite (kg)
XMU	1	DP	O	Gravitational constant for the central body (μ)
XMUI	1	DP	O	$1/\mu$
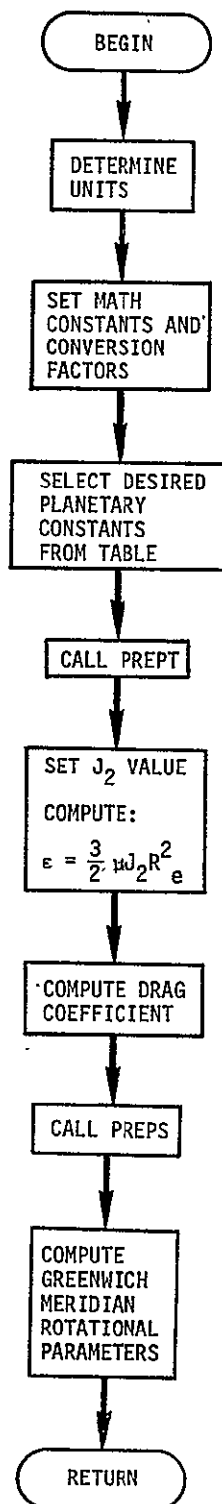
Figure 13.- CONST flow chart.

.3.8 DENSTY (Subroutine)

urpose: Compute the atmospheric density at a given altitude above an oblate
Earth (ref. 15)

alling sequence: CALL DENSTY (ALT,RHO)

alled by: PREPD

ubroutines/functions used: None

amed COMMON: /CARTC /   X,Y,Z,V(3),TIME,ENERGY,R,RI
/CBASIC/   PI,TWOPI,DEG,RAD,DAY,DTOKM
/DATMOS/   FBAR,XKP,SLDAY,SADAY
/DATMO1/   SRAB,CRAB,SDEC,CDEC,RB,TC,TG
./DCOEFF/   A(3,3,3),B(3,9),C(3,9),D(3,4)

rogram data: Size = 672_8 (442_{10}) words compiled

ORTRAN ariable	Dimension	Type	Input/ output	Description
	(3,3,3)	DP	I	Parameters for determining the base altitude
LT	1	DP	I	Altitude above an oblate Earth
	(3,9)	DP	I	Parameters for determining the T_∞ density profile
	(3,9)	DP	I	Parameters for computing annual variation
DEC	1	DP	I	Cosine of bulge declination
RAB	1	DP	I	Cosine of bulge right ascension
	(3,4)	DP	I	Parameters for computing the seasonal latitudinal variation
ΓOKM	1	DP	I	Converts distance into kilometers
	1	DP	I	Magnitude of the position vector of the satellite
3	1	DP	I	Magnitude of the diurnal change in the exospheric temperature
IO	1	DP	O	The computed atmospheric density (kg/m^3)

FORTRAN variable	Dimension	Type	Input/ output	Description
RI	1	DP	I	Inverse R
SADAY	1	DP	I	Magnitude of the semiannual density variation
SDEC	1	DP	I	Sine of the bulge declination
SLDAY	1	DP	I	Magnitude of the seasonal latitudinal density variation
SRAB	1	DP	I	Sine of the bulge right ascension
TC	1	DP	I	Nighttime minimum of the global exospheric temperature (^{O}K)
TG	1	DP	I	Variation in the exospheric temperature due to geomagnetic activity (^{O}K)
X	1	DP	I	Cartesian coordinates for the position of the satellite
Y	1	DP	I	
Z	1	DP	I	
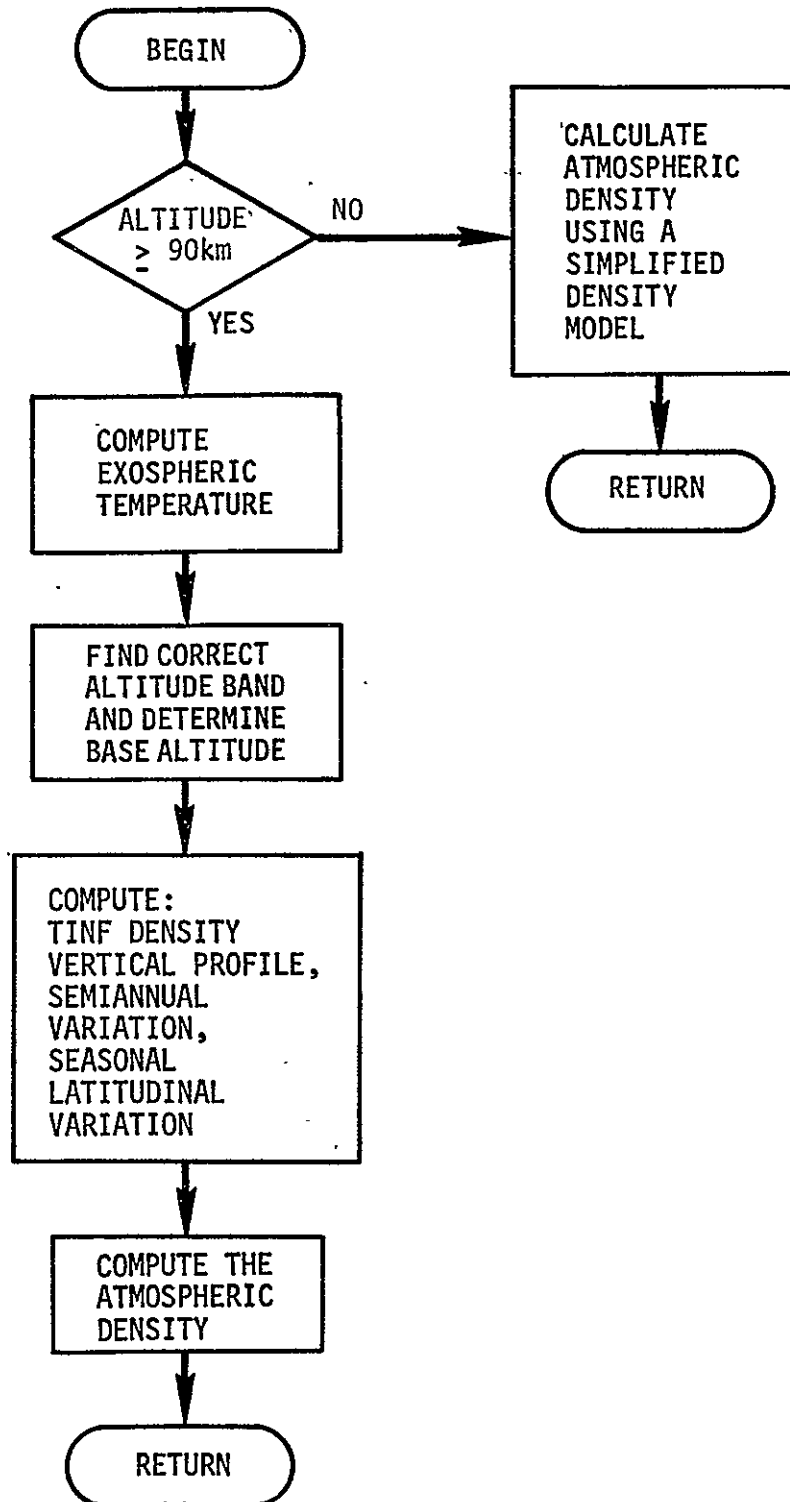
Figure 14.- DENSTY flow chart.

3.3.9 DETERM (Subroutine)

Purpose: Compute the first-order, long-period generating functions and its derivatives (ref. 8)

Calling sequence: CALL DETERM

Called by: LONGPP

Subroutines/functions used: ILOG10

Named COMMON:
/DETE	/	SHAT,SHATP,SHATE2,SHATB,SHATXI,SHATPI
/ECC	/	ES,ESSQ
/EXPCOF/		A(200),B(200),NDEXO,NDEX(18)
/GEO	/	RE,CS2,CS(187),SS(187)
/PSANS2/		SUM2,SUM3,DIFF2,DIFF3,GC,HC,PSSQRT,PS,QS
/RETRO	/	IRO.
/RPOOL	/	XO,X(18),YO,Y(18),PESSQO,PESSQ(9),KMMB2,KMMB21
/S1STAV/		GIN,HOG,GPH,BS,FS,GINSQ
/TESS	/	NMAX,MMAX
/XIPSI	/	XI1,PSI1

Program data: Size = 1467_8 (823_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
A	200	DP	I	Array containing the Binomial coefficients
B	200	DP	I	Array containing the Fourier coefficients for cosine and sine raised to a power
BS	1	DP	I	$b = 1 - H/G$
CS	187	DP	I	C coefficients of the geopotential model in the unnormalized form
EB	1	DP	O	$e \sqrt{b}$
ES	1	DP	I	e
ESSQ	1	DP	I	e^2
IESSQ	1	I	I	
IMMAX	1	I	I	Number of terms to be generated for the expansion of the Earth's geopotential model
INMAX	1	I	I	

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FORTRAN variable	Dimension	Type	Input/ output	Description
IRO	1	I	I	Flag to determine if the orbit is retrograde = -1 yes = 1 no
NDEX	18	I	I	Array of pointers to the A and B coefficients
NMAX	1	I	I	Maximum number of zonal terms to be included by the geopotential model; maximum value of IMMAX and INMAX
NMAX2	1	I	O	Maximum value of IESSQ; (NMAX + 1)/2
PS	1	DP	I	ρ
PSI1	1	DP	I	e sin I cos g
RE	1	DP	I	Central body equatorial radius (R_e)
REOP	1	DP	O	R_e/ρ
SHAT	1	DP	I/O	First-order, long-period generating function
SHATB	1	DP	I/O	
SHATE2	1	DP	I/O	
SHATP	1	DP	I/O	Derivatives of first-order, long-period generating function (SHAT)
SHATPI	1	DP	I/O	
SHATXI	1	DP	I/O	
XI1	1	DP	I	e sin I sin g

54

```
                    ╭─────────────╮
                    │    BEGIN    │
                    ╰─────────────╯
                           │
                           ▼
         ┌───────────────────────────────────┐
         │  COMPUTE FIRST ORDER              │
         │  LONG PERIOD GENERATING           │
         │                      ^            │
         │  FUNCTION S AND ITS               │
         │  DERIVATIVES                      │
         │                                   │
         │   ∂Ŝ   ∂Ŝ   ∂Ŝ    ∂Ŝ    ∂Ŝ        │
         │   ──   ──   ──    ──    ──        │
         │   ∂p,  ∂b,  ∂e²,  ∂Ψ₁,  ∂x₁.      │
         └───────────────────────────────────┘
                           │
                           ▼
                    ╭─────────────╮
                    │   RETURN    │
                    ╰─────────────╯
```

COMPUTE FIRST ORDER LONG PERIOD GENERATING FUNCTION $\hat{S}$ AND ITS DERIVATIVES

$$\frac{\partial \hat{S}}{\partial p}, \frac{\partial \hat{S}}{\partial b}, \frac{\partial \hat{S}}{\partial e^2}, \frac{\partial \hat{S}}{\partial \Psi_1}, \frac{\partial \hat{S}}{\partial x_1}.$$

Figure 15.- DETERM flow chart.

3.3.10 DRAG (Subroutine)

Purpose: Adjust the PS elements to account for the atmospheric drag pertur-
bations (refs. 6 and 7)

Calling sequence: CALL DRAG

Called by: PSANS

Subroutines/functions used: None

Named COMMON: /DRAG1 /   CFORCE(8),T4,TEMPO
               /PS   /   SIG(4),RHO(4),TAU

Program data: Size = 115_8 (77_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
CFORCE	8	DP	I	Drag force defined in PS elements
RHO	4	DP	I/O	PS elements $\rho_1,\rho_2,\rho_3,\rho_4$
SIG	4	DP	I/O	PS elements $\sigma_1,\sigma_2,\sigma_3,\sigma_4$
TAU	1	DP	I	Independent variable of the PS elements (τ)
TEMPO	1	DP	I	Second-order correction for density due to drag
TLINER	1	DP	I	Change in time due to drag
T4	1	DP	I	Magnitude of the quadratic variation in the mean anomaly

```
         ┌─────────────┐
         │    BEGIN     │
         └──────┬──────┘
                │
                ▼
        ┌───────────────┐
        │  ADJUST PS    │
        │  ELEMENTS     │
        │ σₙ AND ρₙ      │
        └───────┬───────┘
                │
                ▼
         ┌─────────────┐
         │   RETURN     │
         └─────────────┘
```
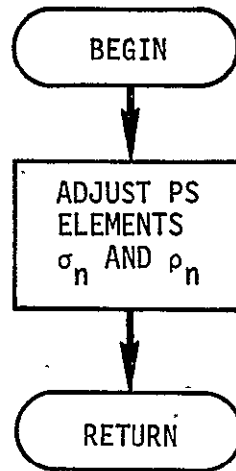
Figure 16.- DRAG flow chart.

3.3.11 FPRIME. (Subroutine)

Purpose: Compute the second-order zonal Hamiltonian and its derivatives (ref. 8)

Calling sequence: CALL FPRIME

Called by: LONGPP

Subroutines/functions used: ILOG10

Named COMMON: /ECC   /   ES,ESSQ
               /EXPCOF/   A(200),B(200),NDEXO,NDEX(18)
               /FP   /   FHAT,FHATP,FHATE2,FHATB
               /GEO   /   RE,CJ2,CS(187),SS(187)
               /PSANS2/   SUM2,SUM3,DIFF2,DIFF3,GC,HC,PSSQRT,PS,QS
               /RPOOL /   YY(30),PESSQO,PESSQ(9),KMMB2,KMMB21
               /S1STAV/   GIN,HOG,GPH,BS,FS,GINSQ
               /TESS   /   NMAX,MMAX

Equivalence: (CS(1),CJ(1)),(SS(1),SJ(1))

Program data: Size = 1177_8 (639_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/output	Description
A	200	DP	I	Array containing the binomial coefficients
B	200	DP	I	Array containing the Fourier coefficients for cosine and sine raised to a power
BS	1	DP	I	b = 1 – H/G
CJ	187	DP	I	C coefficients of the geopotential model in the unnormalized form
ESSQ	1	DP	I	e^2
FHAT	1	DP	I/O	Second-order zonal Hamiltonian
FHATB	1	DP	I/O	Derivatives of second-order zonal Hamiltonian
FHATE2	1	DP	I/O	Derivatives of second-order zonal Hamiltonian
FHATP	1	DP	I/O	Derivatives of second-order zonal Hamiltonian

FORTRAN variable	Dimension	Type	Input/ output	Description
IESSQ	1	I	I ⎫	Number of terms to be gen-erated for the expansion of
INMAX	1	I	I ⎭	the geopotential model
NDEX	1	I	I	Array of pointers to the A and B coefficients
NMAX	1	I	I	Maximum number of zonal terms to be included by the geopotential model; maximum value of INMAX
NMAX2	1	I	O	Maximum value of IESSQ; (NMAX + 1)/2
PS	1	DP	I	p
RE	1	DP	I	Central body equatorial ra-dius (R_e)
REOP	1	DP	O	R_e/p
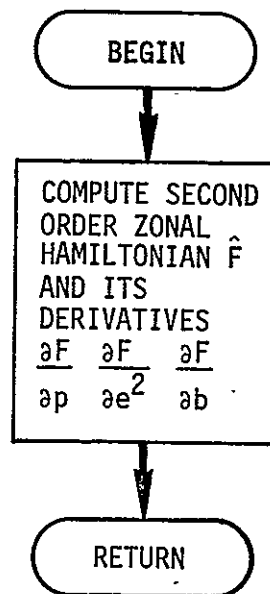
Figure 17.- FPRIME flow chart.

3.3.12 GEOPOT (Subroutine)

Purpose: Compute the Earth's geopotential in a recursive manner (ref. 16)

Calling sequence: CALL GEOPOT (POT)

Called by: PSTOX, XTOPS

Subroutines/functions used: None

Named COMMON: /CARTC /   X,Y,Z,VX(3),TIME,ENERGY,R,RI
               /CBASIC/   PI,TWOPI,DEG,RAD,DAY,DTOKM
               /CBODY /   XMU,XMUI,SQTMU,SQTMUI,EPS
               /GEO   /   RE,CJ2,CS(187),SS(187),IGEOFL
               /GMTROT/   WE,THETAO
               /PERTRB/   IDRAG,ILONG
               /RPOOL /   POO,PO(19),P10,P1(19),P20,P2(19),CTILO,CTIL(19),
                          STILO,STIL(19),R2I,R3I
               /TESS /   NMAX,MMAX

Equivalence: (CTIL(1),CTIL1),(STIL(1),STIL1)

Program data: Size = 430_8 (280_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
CS	187	DP	I	C coefficient of the geopotential model in the unnormalized form
EPS	1	DP	I	$= 3/2 \ (\mu \ J_2 \ R_e^2)$
ILONG	1	I	I	Flag to determine the type of geopotential terms to be included
				= 0 none (two-body orbit)
				= 1 J_2 short period, and first-order secular terms
				= 2 Compute the mean energy due to the geopotential terms as defined by NMAX and MMAX

FORTRAN variable	Dimension	Type	Input/ output	Description
MMAX	1	I	I	Total number of tesseral terms to be included by the geopotential model (value is needed only when ILONG = 2)
POT	1	DP	O	Magnitude of the Earth's gravitational potential
R	1	DP	I	Magnitude of the position vector of the satellite
RE	1	DP	I	Equatorial radius of the central body (Earth)
RI	1	DP	I	Inverse of R
SS	187	DP	I	S coefficients of the geopotential model in the unnormalized form
THETAO	1	DP	I	Initial hour angle of the Earth
TIME	1	DP	I	Elapsed time
TWOPI	1	DP	I	2π
WE	1	DP	I	Rotational rate of the Earth
X	1	DP	I	X-Component of the Earth inertial position vector
XMU	1	DP	I	μ
Y	1	DP	I	Y-Component of the Earth inertial position vector
Z	1	DP	I	Z-Component of the Earth inertial position vector
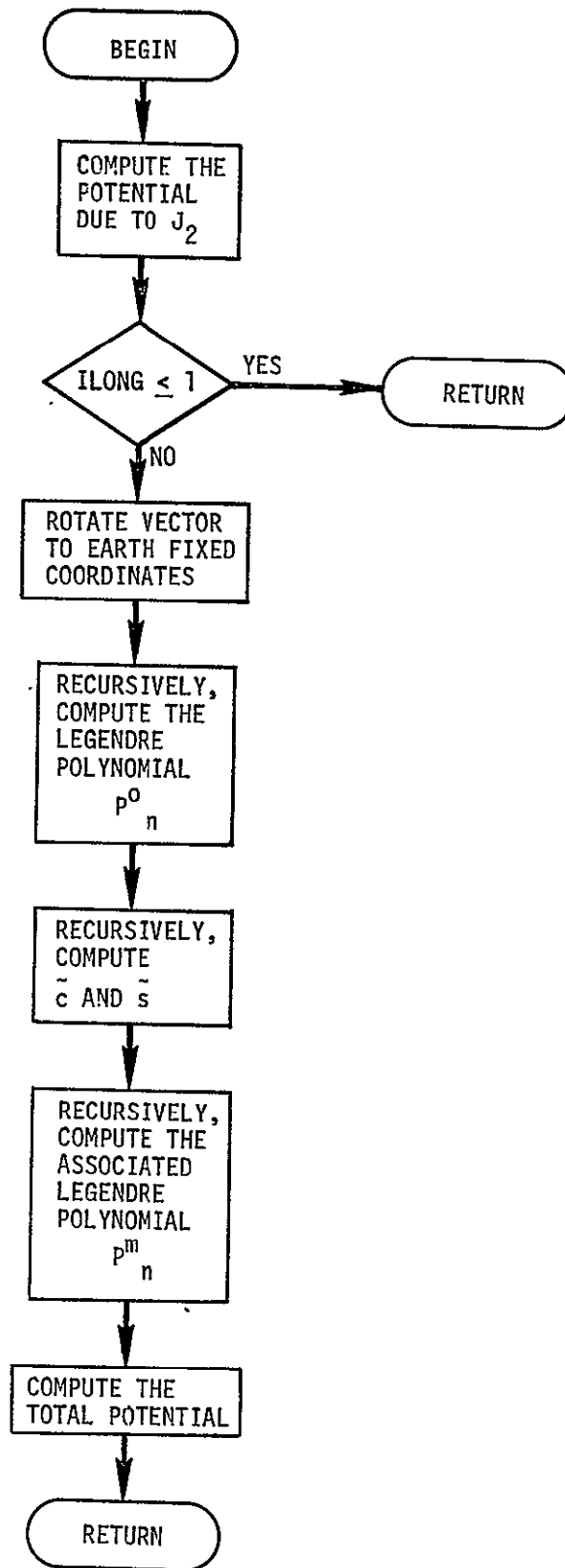
Figure 18.- GEOPUT flow chart.

3.3.13 ILOG10 (Subroutine)

Purpose: Determine the number of terms to be included in the expansion for the Earth's geopotential model

Calling sequence: CALL ILOG10 (X,MAX,IMAX)

Called by: DETERM,FPRIME,POTEXP

Subroutines/functions used: none

Named COMMON: none

Program data: Size = 106_8 (70_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
IMAX	1	I	O	Number of terms, between 1 and MAX, to be included in the expansion for Earth's geopotential model
MAX	1	I	I	Maximum value of IMAX
X	1	DP	I	Small parameter used in determining number of terms to be included in the expansion
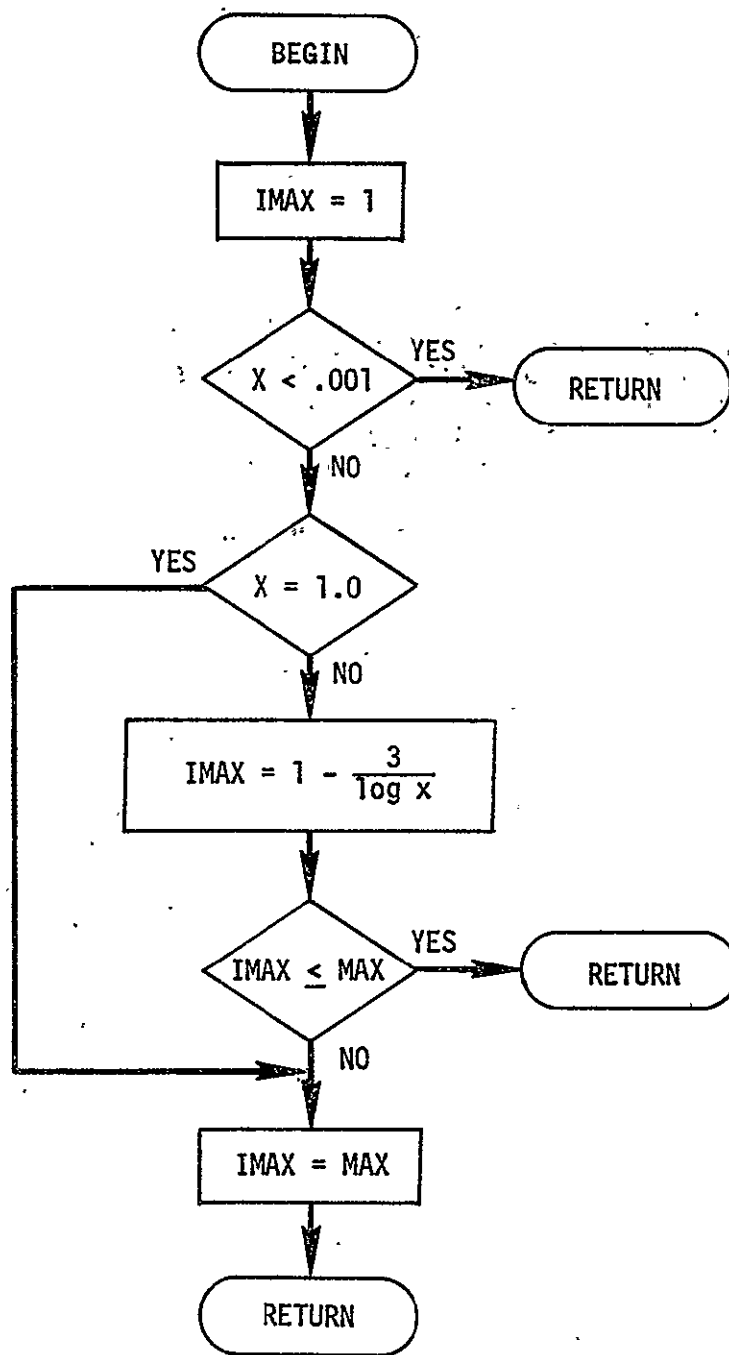
Figure 19.- ILOG10 flow chart.

3.3.1# INITAL

Purpose: Initialize the coefficients for the Jacchia 71/Lineberry atmospheric density model

Calling sequence: CALL INITAL (TIME)

Called by: PREPD

Subroutines/functions used: SACT, SUN

Named COMMON:

/CBASIC/	PI,TWOPI,DEG,RAD,DAYSEC,DTOKM
/DATMOS/	FBAR,AKP,SLDAY,SADAY
/DATMO1/	SRAB,CRAB,SDEC,CDEC,RB,TC,TG
/DCOEFF/	A(3,3,3),B(3,9),C(3,9),D(3,4)
/EPOCH /	CDATE(6),XJDATE
/SUNPOS/	X1S,X2S,X3S,RS,RAS,DECS

Program data: Size = 255_8 (173_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
A	(3,3,3)	DP	O	Parameters for determining the base altitude
AKP	1	DP	I	Averaged value for the geomagnetic index
B	(3,9)	DP	O	Parameters for determining the T_∞ density profile
C	(3,9)	DP	O	Parameters for computing annual variation
CDEC	1	DP	O	Cosine of bulge declination
CRAB	1	DP	O	Cosine of bulge right ascension
D	(3,4)	DP	O	Parameters for computing the seasonal latitudinal variation .
DAY	1	DP	O	Julian day number for which the solar activity coefficients are desired
DAYSEC	1	DP	I	Converts days into seconds, minutes, or hours

FORTRAN variable	Dimension	Type	Input/ output	Description
FBAR	1	DP	I	Averaged value for the solar flux coefficient, $\overline{F}_{10.7}$
RAD	1	DP	I	$180/\pi$
RAS	1	DP	I	Right ascension of the Sun
RB	1	DP	O	Magnitude of the diurnal change in the exospheric temperature
RS	1	DP	I	Magnitude of position vector of the Sun
SADAY	1	DP	O	Magnitude of the semiannual density variation
SDEC	1	DP	O	Sine of bulge declination
SLDAY	1	DP	O	Magnitude of the seasonal latitudinal density variation
SRAB	1	DP	O	Sine of bulge right ascension
TC	1	DP	O	Night time minimum of the global exospheric temperature ($^{\circ}K$)
TG	1	DP	O	Variation in the exospheric temperature due to geomagnetic activity ($^{\circ}K$)
TIME	1	DP	I	Elapsed time of epoch
TWOPI	1	DP	I	2π
XJDATE	1	DP	I	Julian date of epoch
X3S	1	DP	I	Position vetor of the Sun in the Earth's inertial equatorial system (X1S, X2S, X3S)
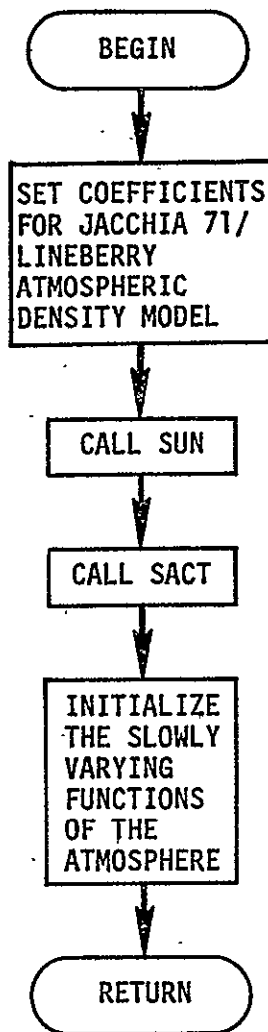
Figure 20.- INITAL flow chart.

3.3.15 INPUT (Subroutine)

Purpose: Read the input data from the NAMELIST statement, set the default
values, and initialize all required COMMON block variables.

Calling sequence: CALL INPUT ($20.)

Called by: MAIN

Subroutines/functions used: AEIXYZ, CDTOJD, CONST, OUTPUT, XTOPS

Named COMMON: /CARTC /   X(6),TIME,ENERGY,R,RI
               /CBASIC /   PI,TWOPI,DEG,RAD,DAY,DTOKM
               /CPRINT /   PRINT,IPRINT,IPSPRT,IUNITS
               /DRAG   /   CD,AREA,XMASS,DRDR
               /END   /   STOP,ISTOP
               /EPOCH /   DATE(6),XJDATE
               /KEPLER /   EL(6)
               /PERTURB/   IDRAG,ILONG
               /TESS   /   NMAX,MMAX

NAMELIST statements: /INPUT/   EL,IEL,STOP,ISTOP,PRINT,IPRINT,DATE,IDRAG,CD,
                                 AREA,XMASS,ILONG,NMAX,MMAX,IPSPRT,IUNITS

Program data: Size = 407_8 (263_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
AREA	1	DP	I	Cross sectional surface area of the satellite (needed only if IDRAG $\geq$ 1) (m^2)
CD	1	DP	I	Coefficient of drag (needed only if IDRAG $\geq$ 1)
DATE	6	DP	I	Calendar date of epoch

DATE(1) = month
     (2) = day number
     (3) = year
     (4) = hour
     (5) = minutes
     (6) = seconds

FORTRAN variable	Dimension	Type	Input/ output	Description
EL	6	DP	I/O	Initial conditions of the satellite given in Keplerian elements of Cartesian coordinates; on output it will contain the Keplerian elements EL(1) = X or a (2) = Y or e (3) = Z or i (4) = X or ω (5) = Y or Ω (6) = Z or M
ENERGY	1	DP	O	Total energy of the satellite; initially set to 0
IDRAG	1	I	I	Flag to determine if the drag calculations are to be included = 0 no = 1 yes
IEL	1	I	I	Flag to determine if the input values of EL are given as Keplerian elements or Cartesian coordinates = 1 Keplerian = 2 Cartesian
ILONG	1	I	I	Flag to determine type of geopotential terms to be included = 0 none (two-body) = 1 J_2 short period, and first-order secular terms = 2 Compute the mean energy due to the geopotential terms as defined by NMAX and MAX

FORTRAN variable	Dimension	Type	Input/ output	Description
IPRINT	1	I	I	Flag to determine if the intermediate printout is to be done at a PRINT value of days or revolutions = 0 no intermediate printout = 1 days = 2 revolutions
IPSPRT	1	I	I	Flag to determine if the PS elements are to be included with all output = 0 no = 1 yes
ISTOP	1	I	I	Flag to determine if the STOP condition is days or revolutions = 1 days = 2 revolutions
IUNITS	1	I	I	Flag to determine what calculation constants are to be used = 1 km,sec = 2 nm,sec = 3 ft,sec = 4 m,sec = 5 km,hr = 6 nm,hr = 7 E.r.,min
MMAX	1	I	I	Maximum number of tesseral terms to be included (needed only if ILONG $\geq$ 2)
NMAX	1	I	I	Maximum number of zonal terms to be included (needed only if ILONG $\geq$ 2)
PRINT	1	DP	I	Increment at which the intermediate printout is desired (needed only if IPRINT $\geq$ 1) (days or revs)

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FORTRAN variable	Dimension	Type	Input/ output	Description
RAD	1	DP	I	$\pi/180$
STOP	1	DP	I	Final stop value at which output is desired (days or revs)
TIME	1	DP	O	Physical time; initially set to 0 (hrs or min or sec)
X	6	DP	O	Initial Cartesian state vector $X(1) \rightarrow X(3) = \vec{X}$ $X(4) \rightarrow X(6) = \vec{V}$
XJDATE	1	DP	I	Julian date
XMASS	1	DP	I	Initial mass of the satellite (kg)

BEGIN

SET DEFAULTS

READ $INPUT
PRINT $INPUT
CALL CDTOJD
CALL CONST

IEL = 2 — YES → X(i) = EL(i)
i = 1,6

NO

DEG. TO RAD.
CALL AEIXYZ

TIME = 0
ENERGY = 0

CALL XTOPS
CALL OUTPUT

INPUT
'X', 'C', or 'S'

X INPUT — YES → RETURN

NO

S INPUT — YES → RETURN 1

NO

SOLICIT FOR
CHANGES TO
INPUT
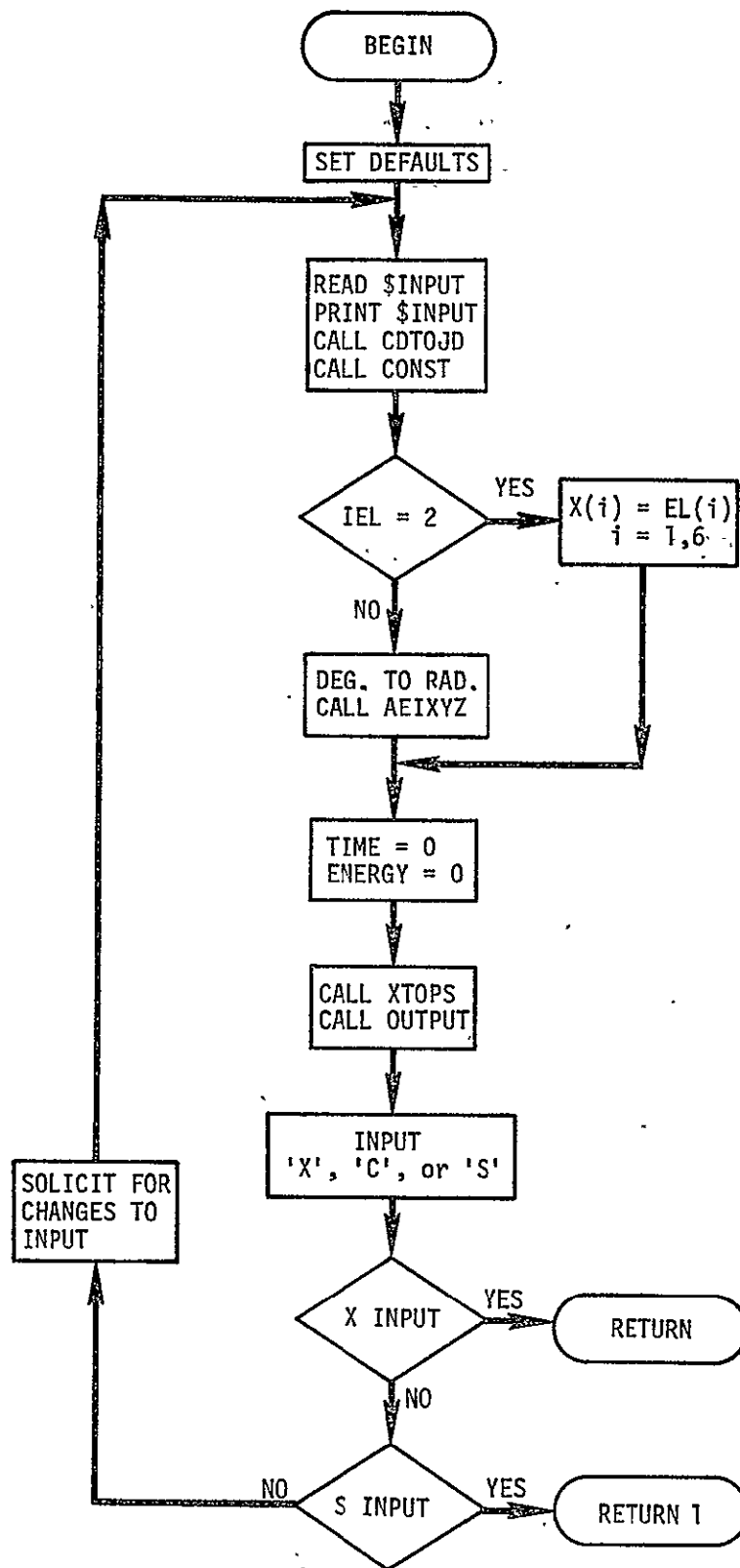
Figure 21.- INPUT flow chart.

3.3.16 JDTOCD (Subroutine)

Purpose: Determine the calendar date corresponding to a given Julian date

Calling sequence: CALL JDTOCD (CDATE,XJDATE).

Called by: OUTPUT

Subroutines/functions used: None

Named COMMON: None

Program data: Size = 160_8 (112_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
CDATE	6	DP	O	The calendar date corresponding to XJDATE, given in the form CDATE(1) = month (2) = day (3) = year (4) = hours (5) = minutes (6) = seconds
XJDATE	1	DP	I	A Julian date
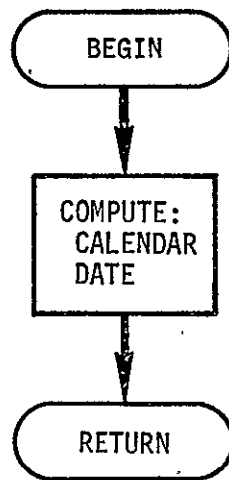
Figure 22.- JDTOCD flow chart.

3.3.17 LONGPP (Subroutine)

Purpose: Compute the first-order, zonal, long periodic perturbations and second-order zonal, secular perturbations (ref. 9)

Calling sequence: CALL LONGPP(NN)

Called by: PSANS

Subroutines/functions used: DETERM, FPRIME

Named COMMON:

Named COMMON:	/CBODY /	XMU,XMUI,SQTMU,SQTMUI,EPS
	/CONSTW/	TWO3,BY3,BY6,CN
	/DENS /	B(10),DS1,DS2,DC1,DC2,OO,OC2,OS2,OC1,OS1
	/DETE /	SHAT,SHATP,SHATE2,SHATB,SHATXI,SHATPI
	/ECC /	ES,ESSQ
	/FP /	FHAT,FHATP,FHATE2,FHATB
	/HAMDS /	DSF(4),DSB(4)
	/PS /	SIG(4),RHO(4),TAU
	/PSANSV/	FACTOR(4),SIGINI(8)
	/PSANS1/	ETA1,ZETA1,TWOL,IQL,FAK
	/PSANS2/	SUM2,SUM3,DIFF2,DIFF3,G,H,PSSQRT,PS,QS
	/PSANS3/	QC
	/RPOOL /	XX(30)
	/S1STAD/	GC(8),P(8),Q(8),HC(8),QCV(8)
	/S1STAV/	GIN,HOG,GPH,BS,FS,GINSQ
	/TESS /	NMAX,MMAX
	/XIPSI /	XI1,PSI1

Equivalence: (L,RHO(4)),(DSB2(1),XX(1)),(DSD(1),XX(5)),(DSP(1),XX(9)),(DSE(1),XX(13)),(DSQ(1),XX(17)),(DSDEL(1),XX(21)),(DSDEL1(1),XX(25))

Program data: Size = 1705_8 (965_{10}) words compiled

A description of the mathematical symbols used and their relationship to one another is given in references 8 and 17; a brief description can also be found in Appendices E and F. Therefore, only a brief mathematical description will be given.

FORTRAN variable	Dimension	Type	Input/ output	Description
BS	1	DP	I	$b = 1 - H/G$
CN	1	DP	I	± 1 depending on value of NN
DSB	4	DP	I	$\partial b/\partial \beta_k$ k = 1, 2, 3, 4
DSF	4	DP	I	$\partial f/\partial \beta_k$ k = 1, 2, 3, 4

FORTRAN variable	Dimension	Type	Input/ output	Description
EPS	1	DP	I	$E = 3/2\ (\mu J_2 R_e^2)$
ES	1	DP	O	e
ESSQ	1	DP	O	e^2
FACTOR	4	DP	O	Derivatives of the DS Hamiltonian and its combinations (A_1, A_2, A_3 and A_4 in Appendix F)
FAK	1	DP	I	$(2L)^{-3/2}$
FHAT	1	DP	I/O	Second-order zonal Hamiltonian
FHATB	1	DP	I/O	Derivatives of second-order zonal Hamiltonian
FHATE2	1	DP	I/O	
FHATP	1	DP	I/O	
FS	1	DP	I	$f = 1/pq$
GC	8	DP	I	$\partial G/\partial\sigma_k,\ \partial G/\partial\rho_k\ k = 1,\ 2,\ 3,\ 4$
GIN	1	DP	I	G^{-1}
GINSQ	1	DP	I	G^{-2}
GPH	1	DP	I	$G + H$
HC	8	DP	I	$\partial H/\partial\sigma_k,\ \partial H/\partial\rho_k$ $k = 1,\ 2,\ 3,\ 4$
HOG	1	DP	I	H/G
L	1	DP	I/O	$L = \rho_4 = \sigma_8$
NMAX	1	I	I	Maximum number of zonal terms to be included
NN	1	I	I	Flag determining if initializing or computing.

= 0 initializing
= 1 computing

FORTRAN variable	Dimension	Type	Input/output	Description
OC1	1	DP	O	Coefficients of Fourier series describing density due to long-period J_2 changes in height[a]
OS1	1	DP	O	
P	8	DP	I	$\partial p/\partial \sigma_k$, $\partial p/\partial \rho_k$ $k = 1, 2, 3, 4$
PS	1	DP	I	p
PSI1	1	DP	O	$\psi_1 = e \sin I \cos g$
PSSQRT	1	DP	I	$\sqrt{p}$
Q	8	DP	I	$\partial q/\partial \sigma_k$, $\partial q/\partial \rho_k$ $k = 1, 2, 3, 4$
QC	1	DP	I	Q
QCV	8	DP	I	$\partial Q/\partial \sigma_k$, $\partial Q/\partial \rho_k$ $k = 1, 2, 3, 4$
QS	1	DP	I	q
RHO	4	DP	I/O	$\rho_1, \ldots, \rho_4$ (see L)
SHAT	1	DP	I/O	First-order, long-period generating function
SHATB	1	DP	I/O	Derivatives of first-order, long-period generating function
SHATE2	1	DP	I/O	
SHATP	1	DP	I/O	
SHATPI	1	DP	I/O	
SHATXI	1	DP	I/O	
SIG	4	DP	I/O	$\sigma_1, \ldots, \sigma_4$
SQTMU	1	DP	I	$\sqrt{\mu}$
SQTMUI	1	DP	I	$1/\sqrt{\mu}$
TWO3	1	DP	I	2/3

[a]Reference 7, pp. 18-22.

FORTRAN variable	Dimension	Type	Input/ output	Description
TWOL	1	DP	I	2L
XI1	1	DP	O	$X_1 = e \sin I \sin g$
XMU	1	DP	I	μ
XMUI	1	DP	I	$1/\mu$
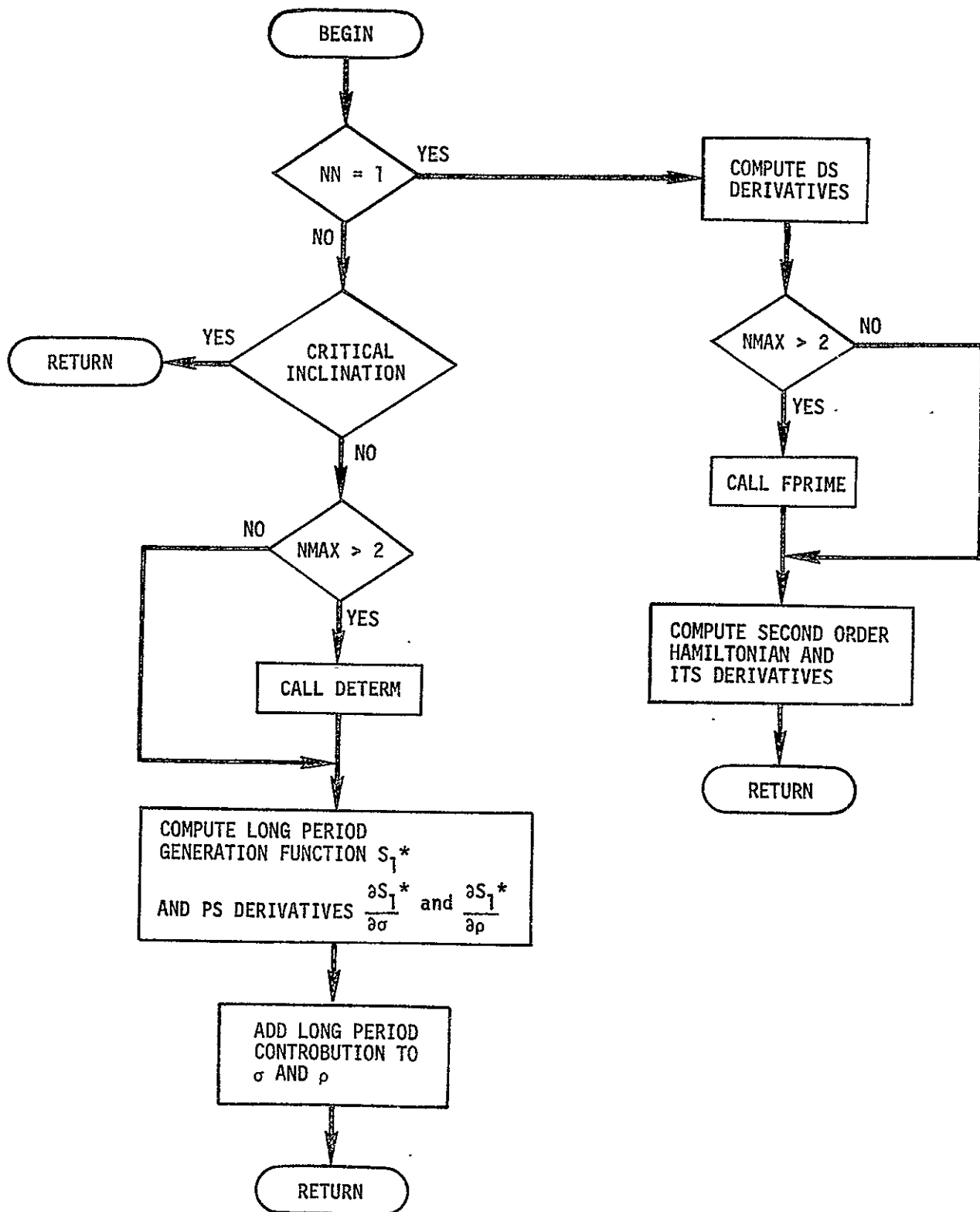
Figure 23.- LONGPP flow chart.

3.3.18 <u>MATIN (Subroutine)</u>

<u>Purpose</u>: Invert an n x n matrix and/or solve the matrix equation Ax = B

<u>Calling sequence</u>: CALL MATIN (A,N,B,M,KEY,DETERM)

<u>Called by</u>: PREPD

<u>Subroutines/functions used</u>: ·None

<u>Named COMMON</u>: None

<u>Program data</u>: Size = 573_8 (379_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
A	(10,10)	DP	I	Matrix to be inverted
B	10	DP	I	Column matrix to be multiplied by A
DETERM	1	DP	O	Determinant
KEY	1	I ·	O	Flag for singular matrix
M	1	I	I	Flag to determine if the B matrix is to be used = 0 no = 1 yes
N	1	I	I	Size of A matrix and length of B
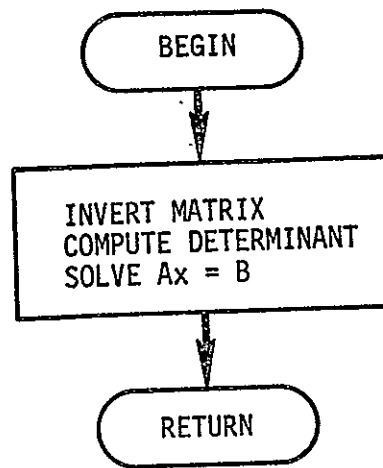
Figure 24.- MATIN flow chart.

3.3.19 MTOECC (Subroutine)

Purpose: Convert mean anomaly to eccentric anomaly and compute its sine and cosine

Calling sequence: CALL MTOECC (XM,E,EA,SINEA,COSEA)

Called by: AEIXYZ, SUN

Subroutines/functions used: None

Named COMMON: None

Program data: Size = 163_8 (115_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
COSEA	1	DP	O	Cosine of the eccentric anomaly (cos E)
E	1	DP	I	Orbital eccentricity (e)
EA	1	DP	O	Eccentric anomaly (rad)
SINEA	1	DP	O	Sine of the eccentric anomaly (sin E)
XM	1	DP	I	Mean anomaly (M)(rad)

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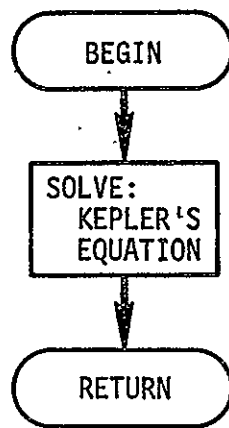
Figure 25.- MTOECC flow chart.

3.3.20 <u>OUTPUT (Subroutine)</u>

<u>Purpose</u>: Print all desired output during the execution of the ASOP program; it
contains all output formats and does all unit conversions required for
output.

<u>Calling sequence</u>: CALL OUTPUT (IFORM)

<u>Called by</u>: INPUT, MAIN

<u>Subroutines/functions used</u>: JDTOCD, XYZAEI

<u>Named COMMON</u>: /CARTC /   X(6),TIME,ENERGY,R,RI
　　　　　　　　 /CBASIC/   PI,TWOPI,DEG,RAD,DAY,DTOKM
　　　　　　　　 /CPRINT/   PRINT,IP,IELPRT,IU
　　　　　　　　 /END /   STOP,ISTOP
　　　　　　　　 /EPOCH /   CDATE(6),XJDATE
　　　　　　　　 /KEPLER/   XKEP(6)
　　　　　　　　 /PS   /   SIG(4),RHO(4),TAU,TAUMAX,TAUINT

<u>Equivalence</u>: (SEC,CDATE(6))

<u>Program data</u>: Size = 710_8 (456_{10}) words compiled.

FORTRAN variable	Dimension	Type	Input/ output	Description
CDATE	6	DP	I	Calendar date of output in the form CDATE(1) = day number (2) = month (3) = year (4) = hours (5) = minutes (6) = seconds
DAY	1	DP	I	Value to convert days into hours, minutes, or seconds
DEG	1	DP	I	$180/\pi$
ENERGY	1	DP	I	Total energy of the physical system
IELPRT	1	I	I	Flag to determine if the PS elements are to be printed = 0 no = 1 yes

FORTRAN variable	Dimension	Type	Input/ output	Description
IFORM	1	I	I	Flag to determine if the initial or final condition message is to be printed. = 1 initial condition message = 2 no message (intermediate print) = 3 final condition message or reentry condition message
IP	1	I	I	Flag to determine what print condition is being used = 1 days = 2 revolutions = 3 reentry
ISTOP	1	I	I	Flag to determine the final stop condition = 1 days = 2 revolutions = 3 reentry
IU	1	I	I	Pointer to the VEL and DST character arrays
RHO	4	DP	I	PS elements $\rho_1 \rightarrow \rho_4$ $RHO(1) = \rho_1$ $RHO(3) = \rho_3$ $RHO(2) = \rho_2$ $RHO(4) = \rho_4$
SIG	4	DP	I	PS elements $\sigma_1 \rightarrow \sigma_4$ $SIG(1) = \sigma_1$ $SIG(3) = \sigma_3$ $SIG(2) = \sigma_2$ $SIG(4) = \sigma_4$
TAU	1	DP	I	Independent variable of the PS elements set (rad)
TAUINT	1	DP	I/O	Initial value of TAU for which the force model is valid. Initially set to 0.
TIME	1	DP	I	Physical time
TWOPI	1	DP	I	2π

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FORTRAN variable	Dimension	Type	Input/ output	Description
X	6	DP	I	Cartesian state vector
				$\dot{X}(1) \rightarrow X(3) = X$
				$X(4) \rightarrow X(6) = V$
XJDATE	1	DP	I	Julian date
XKEP	6	DP	I/O	Keplerian elements

$$
\begin{aligned}
XKEP(1) &= a \\
(2) &= e \\
(3) &= i \\
(4) &= \omega \\
(5) &= \Omega \\
(6) &= M
\end{aligned}
$$
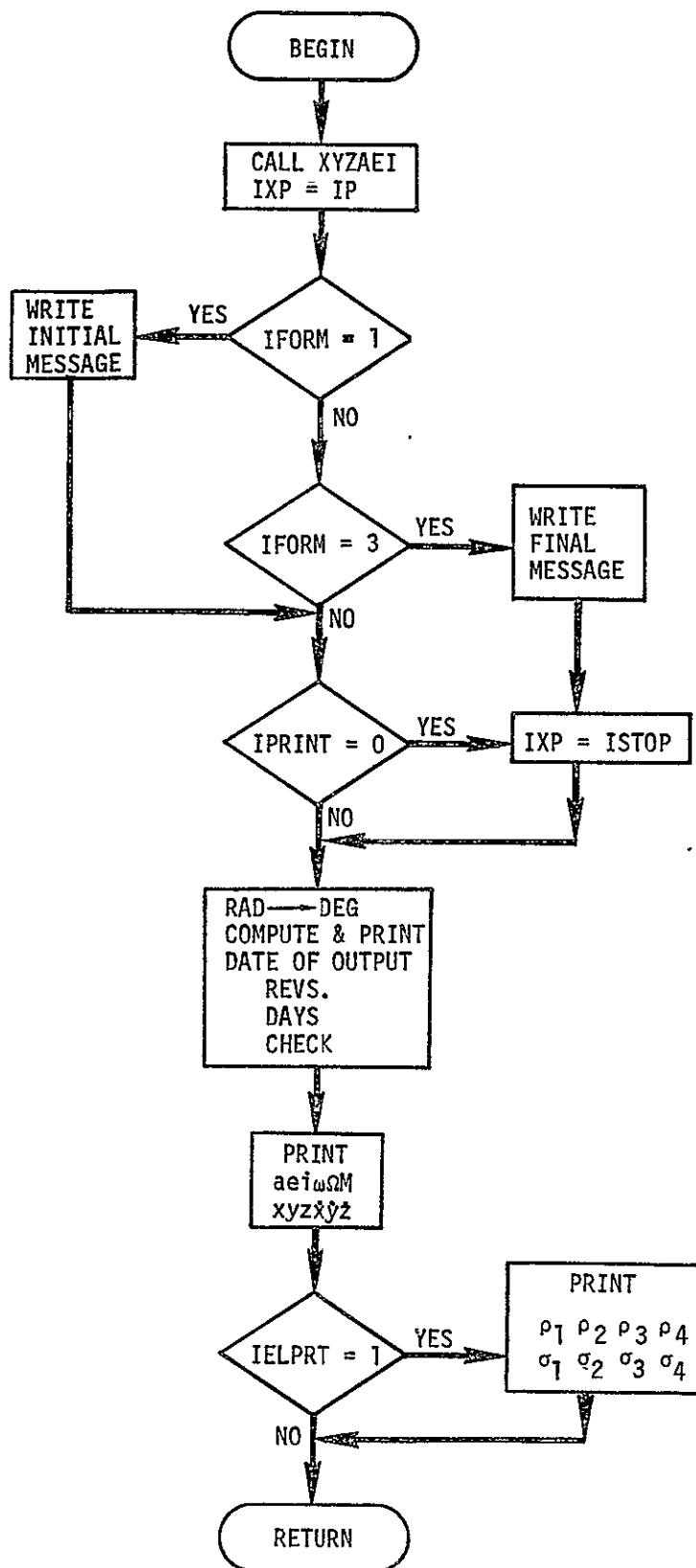
Figure 26.- OUTPUT flow chart.   Page 1 of 1

3.3.21 POTEXP (Subroutine)

<u>Purpose</u>: Compute the mean energy due to tesseral and sectorial geopotential harmonics (ref. 10)

<u>Calling sequence</u>: CALL POTEXP

<u>Called by</u>: ASOP

<u>Subroutines/functions used</u>: COEFF,ILOG10,RECUR,TIMEXP

<u>Named COMMON</u>:

/CBASIC/	PI,TWOPI,DEG,RAD,DAY,DTOKM
/CBODY /	XMU,XMUI,SQTMU,SQTMUI,EPS
/EXPCOF/	A(200),B(200),NDEXO,NDEX(18),IEXPFL
/GEO /	RE,CJ2,CC(187),SS(187),IGEOFL
/GMTROT/	WE,THETAO
/PS /	SIG(4),RHO(4),TAU
/PSANS1/	SSIG1,CSIG1,TWOL,XIQL,FAK
/PSANS2/	SUM2,SUM3,DIFF2,DIFF3,GC,HC,PSSQRT,PS,QS
/PSANS3/	QCAP
/RETRO /	IRO
/RPOOLA/	FNO(24),FN(19,19),FN10(20),FN1(19,19),FM20(20),
	FN2(19,19),BETAO,BETA(18),DELTAO,DELTA(18),
	ETA(20),ZETAO,ZETA(20),ALPHO,ALPH(36),GAMMO,GA
	XIO,XI(38),PSIO,PSI(38),GCAPO,GCAP(17)
/TESS /	NMAX,MMAX

<u>Program data</u>: Size = 2214_8 (1164_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
A	200	DP	I	Array containing binomial coefficients
ALPH	37	DP	I/O	Array generated recursively, where ALPH(1) = $P \rho_3$
B	200	DP	I	Array containing Fourier coefficients
BETA	19	DP	I/O	Array generated recursively, where BETA(1) = $\cos(WE \cdot \sigma_4 + THETAO)$
BS	1	DP	O	$\sqrt{1 - (H/G)^2}$
CC	187	DP	I	C coefficients of the geopotential model in the unnormalized form

FORTRAN variable	Dimension	Type	Input/ output	Description
CSIG1	1	DP	I	$\cos \sigma_1$
DELTA	19	DP	I/O	Array generated recursively, where DELTA(1) = $\sin(WE \cdot \sigma_4 + THETA0)$
ECC	1	DP	O	e
ECC2	1	DP	O	e^2
ETA	21	DP	I/O	Array generated recursively, where ETA(1) = $-Q \, \sigma_2$
GAMM	37	DP	I/O	Array generated recursively where GAMM(1) = $-P \, \sigma_3$
GC	1	DP	I	G
HC	1	DP	I	H
IEXPFL	1	I	I	Flag to determine if A and B arrays have been computed = 0 no = 1 yes
INMAX	1	I	I/O	Number of terms to be generated in BETA, DELTA and XM
IQMAX	1	I	I/O	Number of terms to be generated in ALPH and GAMM
IRMAX	1	I	I/O	Number of terms to be generated in ZETA and ETA
IRO	1	I	I	Flag to determine if the orbit is retrograde = -1 yes = 1 no
ISMAX	1	I	O	Number of terms to be generated in PSI and XI
MAX	1	I	O	Maximum value of IRMAX

FORTRAN variable	Dimension	Type	Input/ output	Description
MMAX	1	I	I	Maximum number of tesseral terms to be included by the geopotential model; maximum value of INMAX; 2*MMAX is maximum value of IQMAX (Needed only when ILONG = 2)
NDEX	18	I	I	Array of pointers to the A and B coefficients
NDEX0	1	I	I	Zero Index to the NDEX array
PS	1	DP	I	p
PSI	39	DP	I/0	Array generated recursively, where $PSI(1) = \cos \sigma_1$
QCAP	1	DP	I	Q
QS	1	DP	I	q
RE	1	DP	I	Equatorial radius of the central body (Re)
REOP	1	DP	O	R_e/p
RHO	4	DP	I/0	PS elements $\rho_1, \rho_2, \rho_3, \rho_4$ Note ρ_4 = energy
SIG	4	DP	I	PS elements $\sigma_1, \sigma_2, \sigma_3, \sigma_4$
SS	187	DP	I	S coefficients of the geopotential model in the unnormalized form
SSIG1	1	DP	I	$\sin \sigma_1$
SUM2	1	DP	I	$\frac{1}{2}(\sigma_2{}^2 + \rho_2{}^2)$
THETA0	1	DP	I	Initial hour angle of the Earth
WE	1	DP	I	Rotational rate of the Earth
XI	39	DP	I/0	Array generated recursively, where $XI(1) = \sin \sigma_1$

FORTRAN variable	Dimension	Type	Input/ output	Description
XIQL	1	DP	I	$\mu/\sqrt{2L}$
XM	(7,18)	DP	O	Fourier coefficients of the time expansion
XMU	1	DP	I	μ
XNU	1	DP	O	Ratio of the frequency of rotation of the satellite to the rotation rate of the Earth
ZETA	21	DP	I/O	Array generated recursively, where $ZETA(1) = Q \rho_2$
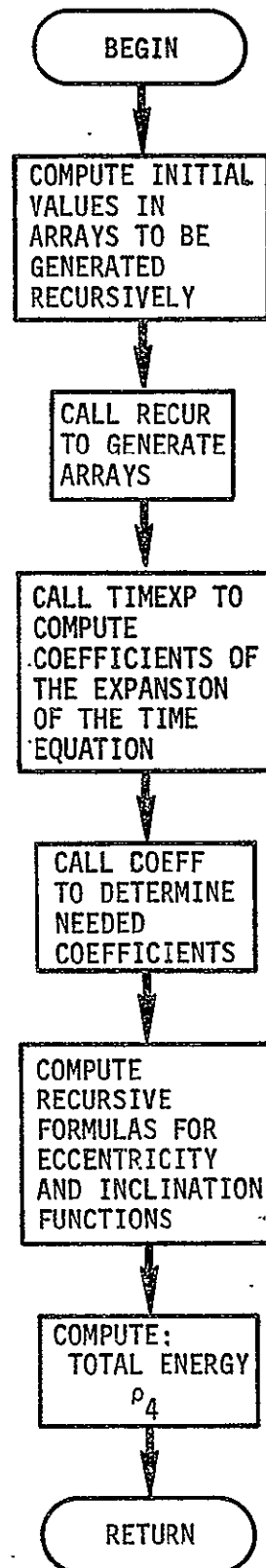
Figure 27.- POTEXP flow chart.

3.3.22 PREPD (Subroutine)

Purpose: Initialize the ASOP atmospheric density model coefficients (refs. 6 and 7)

Calling sequence: CALL PREPD
Called by: ASOP

Subroutines/functions used: CANFOR,DENSTY,INITAL,MATIN,PSTOX,TABLE

Named COMMON:

/CARTC /	X(6),TIME,ENERGY,R,RI
/CBASIC/	PI,TWOPI,DEG,RAD,DAY,DTOKM
/CBODY /	XMU,XMUI,SQTMU,SQTMUI,EPS
/DBETAS/	BETA1,BETA2,BETA3,BETA4
/DENS /	B(10),DS1,DS2,DC1,DC2,OO,OC2,OS2,OC1,OS1
/DRAG /	CD,AREA,XMASS,CDRAG
/DRAG1 /	CFORCE(8),T4,TTL,TEMPO
/END /	STOP,ISTOP
/FORSAV/	Z(6),ZC(9,6),ZS(9,6),DZ(6),DZC(9,6),DZS(9,6)
/GEO /	RE,CJ2,CS(187),SS(187),IGEOFL
/GMTROT/	WE,THETAO
/PERTRB/	IDRAG,ILONG
/PS /	SIG(4),RHO(4),TAU,TAUMAX
/PSANSV/	FACTOR(4),SIGINI(8)
/PSANS1/	SIN1,COS1,TWOL,XIQL,FAK
/PSANS2/	SUM2,SUM3,DIFF2,DIFF3,G,H,PSSQRT,PS,QS
/PSANS3/	QC
/RETRO /	IRO
/SUNPAR/	XNS
/SUNPOS/	XS,YS,ZS,RS,RAS,DECS

Program data: Size = 2212_8 (1162_1) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
ALT	1	DP	O	Calibration altitude for the density model
ALTO	1	DP	O	Reference altitude for the density model
B	1	DP	O	Coefficient of the density model (ref. 7)
BETA1	1	DP	O	β
BETA2	1	DP	O	β^2
BETA3	1	DP	O	β^3

FORTRAN variable	Dimension	Type	Input/ output	Description
BETA4	1	DP	O	β^4
CDRAG	1	DP	I	Drag coefficient
CFORCE	8	DP	I/O	Drag force defined in PS elements
COS1	1	DP	I/O	$\cos \sigma_1$
DC1	1	DP	O	Coefficients of the Fourier series describing the
DC2	1	DP	O	diurnal bulge (ref. 7)
DECS	1	DP	I	Declination of the Sun
DIFF3	1	DP	I	$\rho_3{}^2 - \sigma_3{}^2$
DS1	1	DP	O	Coefficients of the Fourier series describing the
DS2	1	DP	O	diurnal bulge (ref. 7)
DTOKM	1	DP	I	Converts distance into kilometers
DZ	6	DP	I	$\hat{X}_0^i$
DZC	(9,6)	DP	I	$\hat{X}_j^i$ Coefficients of Fourier series expansion of the disturbing function in PS
DZS	(9,6)	DP	I	$\hat{\Psi}_j^i$ elements (ref. 7)
FACTOR	4	DP	I	Derivatives of DS Hamiltonian and its combinations (A_1, A_2, A_3 and A_4 in appendix F)
FAK	1	DP	I	$(2L)^{-3/2}$
G	1	DP	I	G
H	1	DP	I	H
IDRAG	1	DP	O	Flag to determine if drag calculations are to be included = 0 no = 1 yes

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FORTRAN variable	Dimension	Type	Input/ output	Description
IRO	1	DP	I	Flag to determine if the orbit is retrograde. $= -1$ yes $= 1$ no
OC1	1	DP	I/O	Coefficients of the Fourier series describing density variation due to J_2 changes in height (ref. 7)
OC2	1	DP	I/O	
OS1	1	DP	I/O	
OS2	1	DP	I/O	
OO	1	DP	I	
PSSQRT	1	DP	I	p
QC	1	DP	I	Q
QS	1	DP	I	q
R	1	DP	I	Magnitude of the position vector of the satellite
RAD	1	DP	I	$180/\pi$
RAS	1	DP	I	Right ascension of the Sun
RE	1	DP	I	Equatorial radius of the central body
RHO	4	DP	I	PS elements $\rho_1, \rho_2, \rho_3, \rho_4$
RHOD1	1	DP	O	Density at the calibrated altitude
RHOD2	1	DP	O	Density at the reference altitude
SIG	4	DP	I/O	PS elements $\sigma_1, \sigma_2, \sigma_3, \sigma_4$
SIN1	1	DP	I/O	$\sin \sigma_1$
SQTMU	1	DP	I	$\sqrt{\mu}$
STOP	1	DP	I	Final stop value at which output is desired

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FORTRAN variable	Dimension	Type	Input/output	Description
SUM3	1	DP	I	$\frac{1}{2}(\sigma_3{}^2 + \rho_3{}^2)$
TAUMAX	1	DP	O	Range of validity for the force model
TEMPO	1	DP	O	Second order correction for density due to drag
TIME	1	DP	I	Elapsed time of epoch
TTL	1	DP	O	Change in time due to drag (Δt)
TWOL	1	DP	I	2L
TWOPI	1	DP	I	2π
T4	1	DP	O	Magnitude of the quadratic variation in the mean anomaly
WE	1	DP	I	Rotational rate of the Earth
XIQL	1	DP	I	$\mu/2L$
XMU	1	DP	I	Gravitational constant of the central body (μ)
XNS	1	DP	I	Mean motion of Sun
Z	6	DP	I	X_o^i
ZC	(9,6)	DP	I	X_j^i Coefficients of Fourier series expansion of the disturbing function in PS elements (ref. 7)
ZS	(9,6)	DP	I	ψ_j^i

Figure 28.- PREPD flow chart.

3.3.23 PREPS (Subroutine)

Purpose: Establish the parameters needed to calculate the position of the Sun (ref. 18)

Calling sequence: CALL PREPS

Called by: CONST

Subroutines/functions used: none

Named COMMON: /CBASIC/   PI,TWOPI,DEG,RAD,DAY,DTOKM
                /EPOCH /   CDATE(6),XJDATE
                /SUNPAR/   XNS,XLSO,A,E,AEROOT,B1(2),B2(2),B3(2)

Equivalence: (CO,B1(1))

Program data: Size = 276$_8$ (190$_{10}$) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
A	1	DP	O	Semimajor axis of the Sun's orbit
AEROOT	1	DP	O	Argument of perigee of the Sun
B1	2	DP	O	Coefficients to transform the position of the Sun from the orbital plane to the mean-of-epoch equatorial reference system.
B2	2	DP	O	
B3	2	DP	O	
DAY	1	DP	I	Converts days into hours, minutes or seconds
DTOKM	1	DP	I	Converts distance into kilometers
E	1	DP	O	Eccentricity of the Sun's orbit
RAD	1	DP	I	180/π
TWOPI	1	DP	I	2π
XJDATE	1	DP	I	Julian day number of the desired epoch
XLSO	1	DP	O	Mean anomaly of the Sun
XNS	1	DP	O	Mean motion of the Sun

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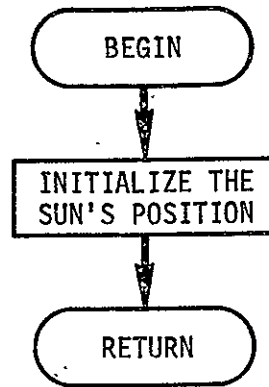
Figure 29.- PREPS flow chart.

3.3.24 PREPT (Subroutine)

Purpose: Initialize the geopotential coefficients for the Earth (ref. 13)

Calling sequence: CALL PREPT

Called by: CONST

Subroutines/functions used: None

Named COMMON: /GEO / RE,CJ2,CS(187),SS(187),IGEOFL
                /RPOOL / FACO,FAC(36)

Program data: Size = 1031_8 (537_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
CS	187	DP	0	C coefficients of the geopotential model in the unnormalized form
IGEOFL	1	I	I/0	Flag to determine if the C and S arrays are set = 0 no = 1 yes
SS	187	DP	0	S coefficients of the geopotential model in the unnormalized form
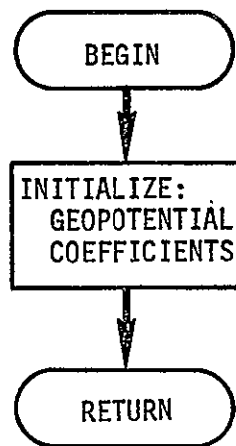
Figure 30.- PREPT flow chart.

3.3.25 PSANS (Subroutine)

Purpose: Analytical theory of the ASOP program.

Calling sequence: CALL PSANS (NN)

Called by: ASOP, TIMEPS

Subroutines/functions used: DRAG,LONGPP

Named COMMON: /CBASIC/ PI,TWOPI,DEG,RAD,DAY,DTOKM
               /CBODY / XMU,XMUI,SQTMU,SQTMUI,EPS
               /CONSTW/ TWO3,BY3,BY6,CN
               /DENS / B(10),DS1,DS2,DC1,DC2,OO,OC2,OS2
               /HAMDS / DSF,DSB
               /PERTRB/ IDRAG,ILONG
               /PS   / SIG(4),RHO(4),TAU
               /PSANSV/ FACTOR(4),SIGINI(8)
               /PSANS1/ ETA1,ZETA1,TWOL,IQL,FAK
               /PSANS2/ SUM2,SUM3,DIFF2,DIFF3,G,H,PSSQRT,PS,QS
               /PSANS3/ QC,EROOT,X3ROOT
               /PSTIME/ CLO,FAKTPS,TOL
               /S1STAD/ GC(8),P(8),Q(8),HC(8),QCV(8)
               /S1STAV/ GIN,HOG,GPH,BS,FS,GINSQ

Equivalences: (LS,SIG(4)), (L,RHO(4)), (PHI,RHO(1)), (DSF(1),W(1)),
               (DSB(1),W(5)), (S1(1),GAM3(1)), (Y(1),GAM2(1)), (Q(1),GAM(1)),
               (HC(1),DEL3(1)), (GC(1),DEL2(1)), (P(1),DEL(1))

Program data: Size = 1712_8 (970_{10}) words compiled

A description of the mathematical symbols used and their relationship to one an-
other is given in reference 17; a brief description can also be found in
Appendixes E and F. Therefore, only a brief mathematical description will be
given.

FORTRAN variable	Dimension	Type	Input/ output	Description
BS	1	DP	O	$b = 1 - H/G$
BY3	1	DP	O	$1/3$
BY6	1	DP	O	$1/6$
CN	1	DP	O	$\pm$ depending on value of NN
DIFF2	1	DP	O	$\rho_2{}^2 - \sigma_2{}^2$
DIFF3	1	DP	O	$\rho_3{}^2 - \sigma_3{}^2$

FORTRAN variable	Dimension	Type	Input/ output	Description
DSB	4	DP	O	$\partial b/\partial_k$ $k = 1, 2, 3, 4$
DSF	4	DP	O	$\partial f/\partial \beta_k$ $k = 1, 2, 3, 4$
EPS	1	DP	I	$\varepsilon = 3/2 \, (\mu \, J_2 \, R_e^2)$
ETA1	1	DP	O	$\sin \sigma_1$
FACTOR	4	DP	I/O	Derivatives of the DS Hamiltonian and its combinations (A_1, A_2, A_3 and A_4 in appendix F)
FAK	1	DP	O	$(2L)^{-3/2}$
FAKTPS	1	DP	O	Derivative of the DS Hamiltonian (A_4 in appendix F)
FS	1	DP	O	$f = 1/pq$
G	1	DP	O	G
GC	8	DP	O	$\partial G/\partial \sigma_k$, $\partial G/\partial \rho_k$ $k = 1, 2, 3, 4$
GIN	1	DP	O	G^{-1}
GINSQ	1	DP	O	G^{-2}
GPH	1	DP	O	$G + H$
H	1	DP	O	H
HC	8	DP	O	$\partial H/\partial \sigma_k$, $\partial H/\partial \rho_k$ $k = 1, 2, 3, 4$
HOG	1	DP	O	H/G
IDRAG	1	I	I	Flag to determine if drag calculations are to be included = 0 no = 1 yes
IQL	1	DP	O	$\mu/\sqrt{2L}$
L	1	DP	I/O	$L = \rho_4 = \sigma_8$

FORTRAN variable	Dimension	Type	Input/ output	Description
LS	1	DP	I/O	$1 = \sigma_4$
NN	1	I	I	Flag determining if initializing or computing = 0 initializing = 1 computing
OO	1	DP	O	Coefficients of Fourier series describing density variation due to J_2 changes in height (ref. 7)
OC2	1	DP	O	
OS2	1	DP	O	
P	8	DP	O	$\partial p/\partial \sigma_k$, $\partial p/\partial \rho_k$ $k = 1, 2, 3, 4$
PHI	1	DP	I/O	$\phi = \rho_1 = \sigma_5$
PS	1	DP	O	p
PSSQRT	1	DP	O	$\sqrt{p}$
Q	8	DP	O	$\partial q/\partial \sigma_k$, $\partial q/\partial \rho_k$ $k = 1, 2, 3, 4$
QC	1	DP	O	Q
QCV	8	DP	O	$\partial Q/\partial \sigma_k$, $\partial Q/\partial \rho_k$ $k = 1, 2, 3, 4$
QS	1	DP	O	q
RHO	4	DP	I/O	$\rho_1, \ldots, \rho_4$ (see SIG)
SIG	4	DP	I/O	$\sigma_1, \ldots, \sigma_4$; note the location of SIG and RHO in COMMON. This location makes the equivalence $\rho_1 = \sigma_5, \ldots, \rho_4 = \sigma_8$
SIGINI	8	DP	O	The initial values of the σ's and ρ's
SQTMU	1	DP	I	$\sqrt{\mu}$
SQTMUI	1	DP	I	$1/\sqrt{\mu}$
SUM2	1	DP	O	$1/2 (\sigma_2^2 + \rho_2^2)$

FORTRAN variable	Dimension	Type	Input/ output	Description
SUM3	1	DP	O	$1/2\ (\sigma_3{}^2 + \rho_3{}^2)$
TAU	1	DP	I	PS Independent variable τ
TWOL	1	DP	O	$2L$
TWO3	1	DP	O	$2/3$
XMU	1	DP	I	μ
XMUI	1	DP	I	μ^{-1}
ZET1	1	DP	O	$\cos \sigma_1$

Figure 31.- PSANS flow charts.

A

COMPUTE:
b AND f

IMARK = 2 → D/4

COMPUTE:
s AND c

CLEAR
WORKING
ARRAY AREA

COMPUTE:
δ_1, δ_2, δ_3
AND
γ_1, γ_2, γ_3

COMPUTE:
DERIVATIVES OF
Q,s,c,G^2-3H^2
$\delta_1,\delta_2,\delta_3,\gamma_1,\gamma_2,\gamma_3,$

COMPUTE:
sin AND cos
OF
$2\sigma_1$, $3\sigma_1$

B/3
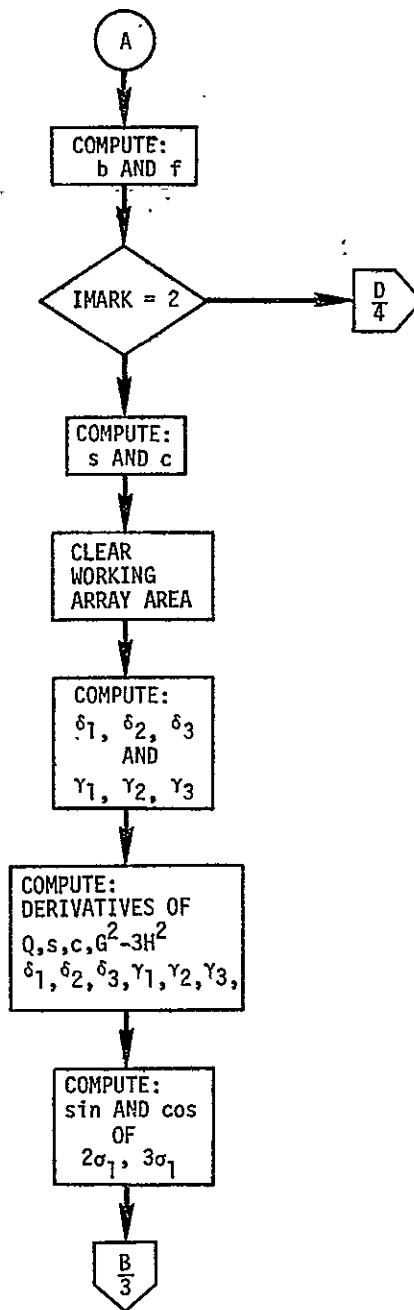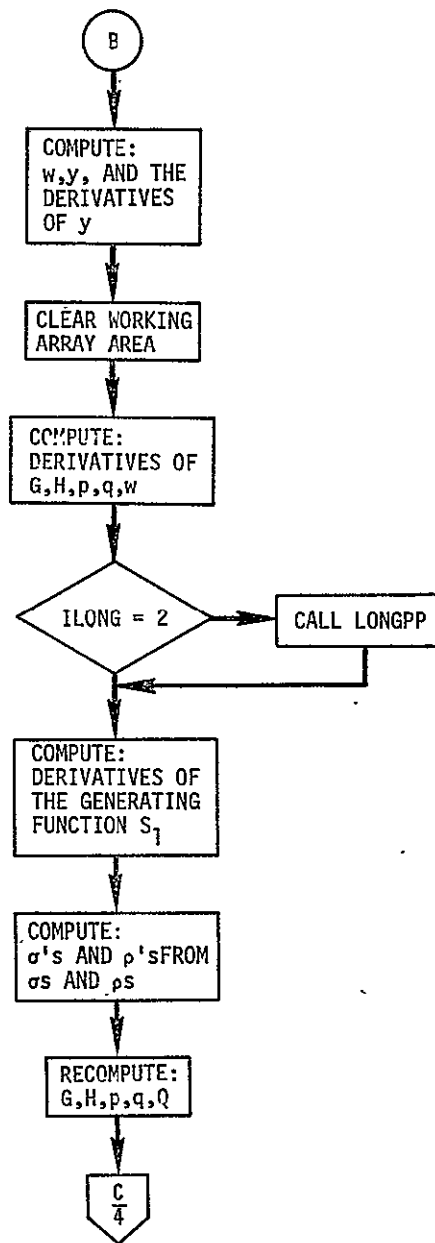
Figure 31.- Continued.

Figure 31.- Continued.

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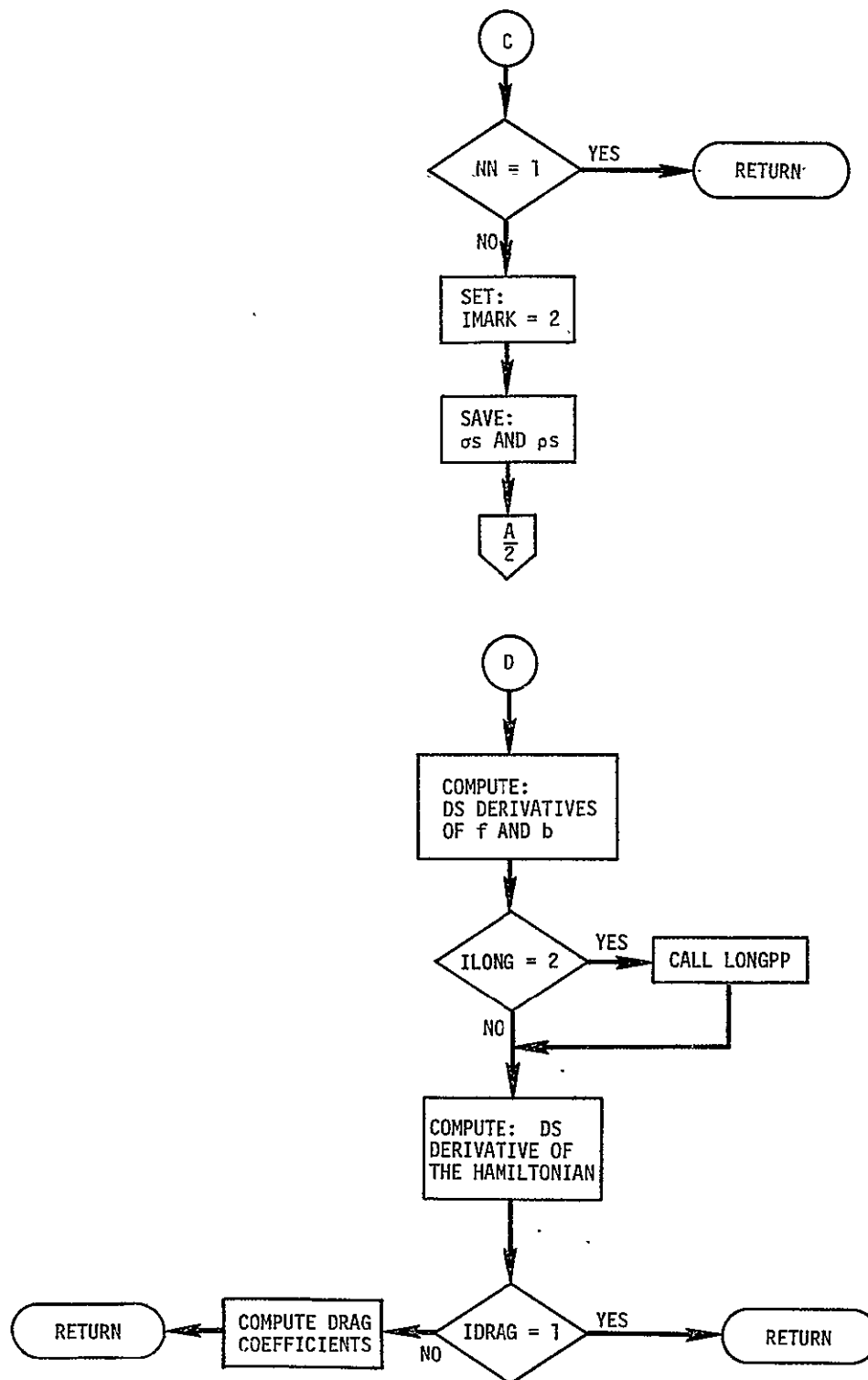
Figure 31.- Concluded.

3.3.26 PSTOX (Subroutine)

Purpose: Transform the PS (Poincaré-Similar) elements into the Cartesian coordi-
nates (X,V); the subroutine will also compute the physical time for
the time iteration stopping procedure.

Calling sequence: CALL PSTOX (ITIME)

Called by: ASOP, PREPD, TIMEPS

Subroutines/functions used: GEOPOT

Named COMMON:

/CARTC /	X1,X2,X3,V1,V2,V3,TIME,ENERGY,R,RI
/CBODY /	XMU,XMUI,SQTMU,SQTMUI,EPS
/PS /	SIG(4),RHO(4),TAU
/PSANS1/	SSIG1,CSIG1,TWOL,XIQL,FAK
/PSANS2/	SUM2,SUM3,DIFF2,DIFF3,GC,HC,PSSQRT,PS,QS
/PSANS3/	QC,EROOT,X3ROOT
/PSTIME/	CLO,FAKPS,TOL
/RETRO /	IRO
/RPOOL /	ECOSPH,ESINPH,ROP,EMINPH,GCIN,RCAP,RDOT,RCAPDT,XXX(6)

Program data: Size = 331_8 (217_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
CS1G1	1	DP	I	$\cos \sigma_1$
ENERGY	1	DP	O	Total energy of the system
EROOT	1	DP	I	$\sqrt{2Lp/\mu}$
FAK	1	DP	I	$(2L)^{-3/2}$
GC	1	DP	I	$\rho_1 - 1/2\,(\sigma_2{}^2 + \rho_2{}^2) = G$
IRO	1	DP	I	Flag to determine if the orbit is retrograde = -1 yes = 1 no

FORTRAN variable	Dimension	Type	Input/output	Description
ITIME	1	I	I	Flag to determine which terms are to be computed
				=0 Compute all Cartesian coordinate elements
				=1 Compute only physical time
				=2 Compute only position vector
				Note: ITIME is assumed to be 0 if it is not 1 or 2
POT	1	DP	I	Magnitude of the Earth's gravitational potential
PS	1	DP	I	p (see section 4.2)
QC	1	DP	I	Q (see section 4.2)
QS	1	DP	I	q (see section 4.2)
R	1	DP	O	Magnitude of the position vector of the satellite
RHO	4	DP	I	$\rho_1,\ldots,\rho_4$
RI	1	DP	O	Inverse magnitude of the position vector of the satellite
SIG	4	DP	I	$\sigma_1,\ldots,\sigma_4.$
SSIG1	1	DP	I	$\sin \sigma_1$
SUM3	1	DP	I	$\sigma_3^2 + \rho_3^2$
TIME	1	DP	O	t
TWOL	1	DP	I	$2\rho_4$
V1	1	DP	O	Components of the velocity vector
V2	1	DP	O	V1 = X component
				V2 = Y component
V3	1	DP	O	V3 = Z component

FORTRAN variable	Dimension	Type	Input/ output	Description
X1	1	DP	O	Components of the position vector
X2	1	DP	O	$X1 = X$ component $X2 = Y$ component
X3	1	DP	O	$X3 = Z$ component
X3ROOT	1	DP	I	$(\sqrt{4G - \sigma_3{}^2 - \rho_3{}^2})/G$
XMU	1	DP	I	μ
XMUI	I	DP	I	μ^{-1}
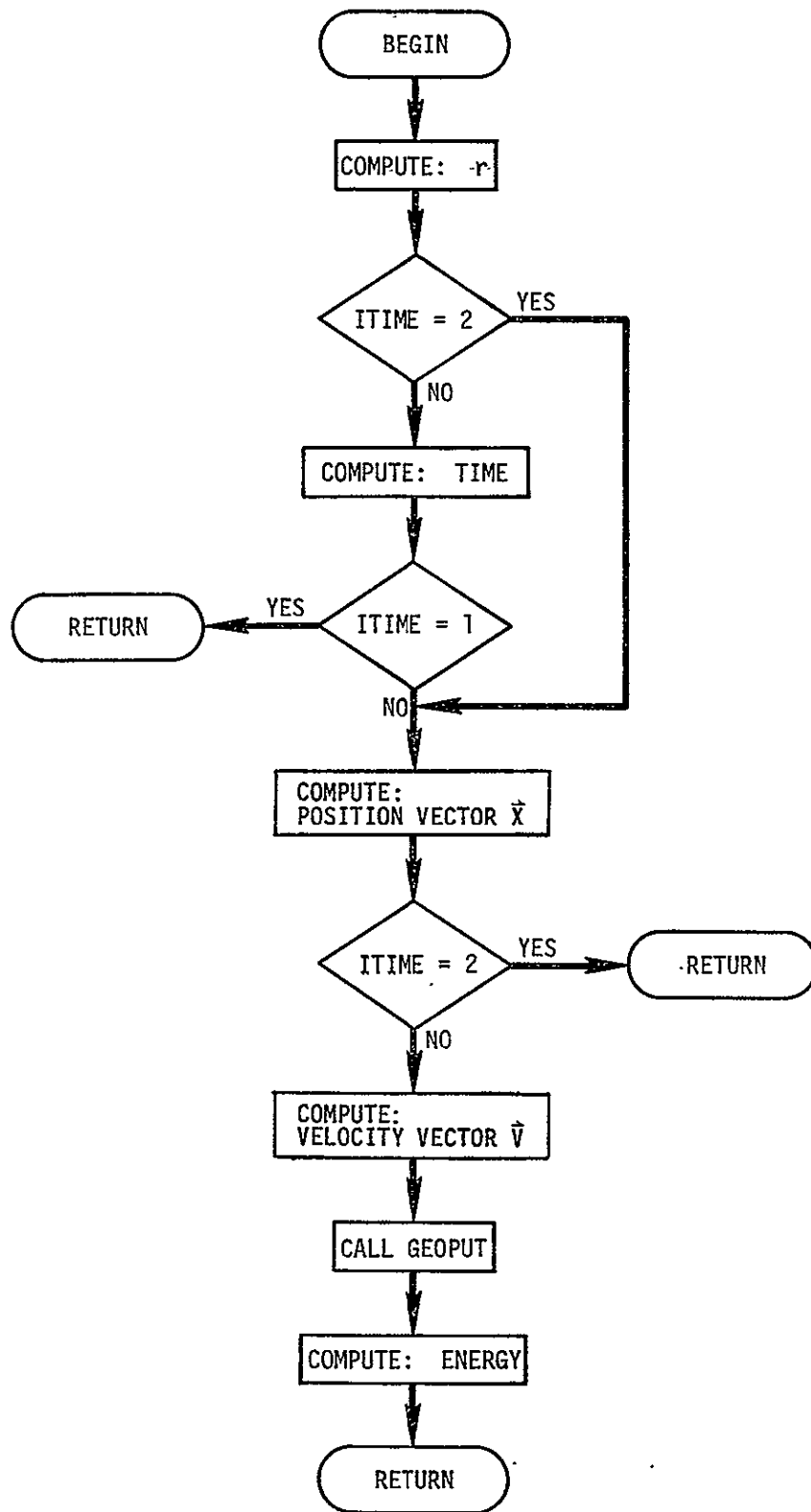
Figure 32.- PSTOX flow chart.

3.3.27 RECUR (Subroutine)

Purpose: Compute, recursively, the sine and cosine of multiples of an angle

Calling sequence: CALL RECUR (COS,SIN,MAX)

Called by: POTEXP

Subroutines/functions used: None

Named COMMON: None

Program data: Size = 163_8 (115_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
COS	MAX	DP	I/O	COS(1) = cosine of initial angle COS = The array $\cos(n\theta)$, n=1,MAX
MAX	1	I	I	Number of terms to be generated
SIN	MAX	DP	I/O	SIN(1) = sine of initial angle SIN = The array $\sin(n\theta)$, n=1,MAX
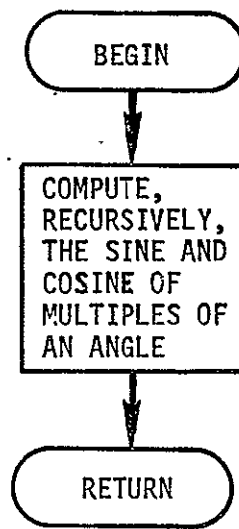
Figure 33.- RECUR flow chart.

3.3.28 SACT (Subroutine)

Purpose: Determine the solar activity coefficients for a given date (ref. 19)

Calling sequence: CALL SACT (DAY, F10BAR, AKP)

Called by: INITAL

Subroutines/functions used: None

Named COMMON: None

Program data: Size = 545_8 (357_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
AKP	1	DP	0	Averaged value for the geomagnetic index
DAY	1	DP	I	Julian day number for which the solar activity coefficients are desired
F10BAR	1	DP	0	Averaged value for the solar flux coefficient, $\overline{F}_{10.7}$
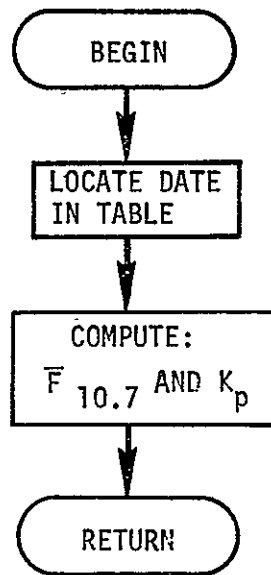
Figure 34.- SACT flow chart.

3.3.29 SUN (Subroutine)

Purpose: Compute the Sun's position analytically (ref. 18)

Calling sequence: CALL SUN (TIME)

Called by: INITAL

Subroutines/functions used: MTOECC

Named COMMON: /CBASIC/ PI,TWOPI,DEG,RAD,DAY,DTOKM
/SUNPAR/ XNS,XLSO,A,E,AEROOT,B1(2),B2(2),B3(2)
/SUNPOS/ XS,YS,ZS,RS,RAS,DECS

Program data: Size = 153_8 (107_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
A	1	DP	I	Semimajor axis of the Sun's orbit
AEROOT	1	DP	I	Argument of perigee of Sun
B1	2	DP	I	Coefficients to transform the position of Sun from orbital plane to mean-of-epoch equatorial reference system
B2	2	DP	I	
B3	2	DP	I	
COSEA	1	DP	I	Cosine of eccentric anomaly
DECS	1	DP	O	Declination of Sun
E	1	DP	I	Eccentricity of Sun
EA	1	DP	I	Eccentric anomaly (rad)
RAS	1	DP	O	Right ascension of Sun
RS	1	DP	O	Magnitude of the position vector of Sun
SINEA	1	DP	I	Sine of eccentric anomaly
TIME	1	DP	I	Elapsed time of epoch
TWOPI	1	DP	I	2π
XLSO	1	DP	I	Mean anomaly of Sun
XM	1	DP	O	Mean anomaly (rad)

FORTRAN variable	Dimension	Type	Input/ output	DESCRIPTION
XNS	1	DP	I	Mean motion of Sun
XS	1	DP	0	Components of the Sun's position vector in the Earth's inertial equatorial system
YS	1	DP	0	
ZS	1	DP	0	

```
        ┌─────────────┐
        │    BEGIN     │
        └─────────────┘
               │
               ▼
        ┌─────────────┐
        │ CALL MTOECC  │
        └─────────────┘
               │
               ▼
    ┌───────────────────────┐
    │                       │
    │  ANALYTICALLY         │
    │  DETERMINE THE        │
    │  SUN'S POSITION       │
    │  ITS MAGNITUDE,       │
    │  THE RIGHT            │
    │  ASCENSION OF         │
    │  THE SUN AND          │
    │  THE DECLINATION      │
    │  OF THE SUN           │
    │                       │
    └───────────────────────┘
               │
               ▼
        ┌─────────────┐
        │   RETURN     │
        └─────────────┘
```
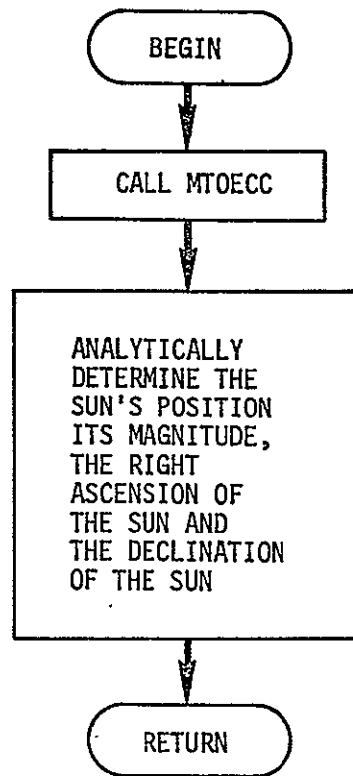
Figure 35.- SUN flow chart.

3.3.30 TABLE (Subroutine)

Purpose: Generate the table of coefficients for the sine and cosine of $n\sigma_1$ for the drag disturbing function (refs. 6 and 7)

Calling sequence: CALL TABLE

Called by: PREPD

Subroutines/functions used: None

Named COMMON: /DBETAS/ B1,B2,B3,B4
/DENS / COEFF(10),DS1,DS2,DC1,DC2,HM,HC2,HS2,HC1,HS1
/DTABLE/ TT(12)
/FORSAV/ Z(6),ZC(9,6),ZS(9,6),DZ(6),DZC(9,6),DZS(9,6)
/PS / SIG(4),RHO(4),TAU

Program data: Size = 1423_8 (787_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
B1	1	DP	I	β
COEFF	10	DP	I	Coefficients of the density model (ref. 7)
DC1	1	DP	I	
DC2	1	DP	I	Coefficients of the Fourier series describing the diurnal
DS1	1	DP	I	bulge
DS2	1	DP	I	
DZ	6	DP	0	$\hat{\chi}_o^i$ Coefficients of the Fourier series expansion of the
DZC	(9,6)	DP	0	$\hat{\chi}_j^i$ disturbing function in PS elements (ref. 7)
DZS	(9,6)	DP	0	$\hat{\psi}_j^i$
HC1	1	DP	I	
HC2	1	DP	I	Coefficients of the Fourier series describing the density variation
HS1	1	DP	I	due to J2 changes in height
HS2	1	DP	I	
RHO	4	DP	I	PS elements ρ_1, ρ_2, ρ_3, ρ_4

FORTRAN variable	Dimension	Type	Input/ output	Description
SIG	4	DP	I	PS elements σ_1, σ_2, σ_3, σ_4
TT	12	DP	0	Table of the averaged Fourier series coefficients
Z	6	DP	0	χ_o^i $\left.\rule{0pt}{1.2em}\right\}$ Coefficients of the Fourier series expansion of the
ZC	(9,6)	DP	0	χ_j^i $\left.\rule{0pt}{1.2em}\right\}$ disturbing function in PS elements (ref. 7)
ZS	(9,6)	DP	0	ψ_j^i $\left.\rule{0pt}{1.2em}\right\}$

Figure 36.- TABLE flow chart.

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3.3.31 TIMEPS (Subroutine)

<u>Purpose</u>: Iteration procedure to stop the PS elements at a desired value of the physical time

<u>Calling sequence</u>: CALL TIMEPS (TFIN,ISET)

<u>Called by</u>: ASOP

<u>Subroutines/functions used</u>: PSANS,PSTOX

<u>Named COMMON</u>:

/CARTC /	X(6),TIMEP,ENERGY,R,RI
/CBASIC/	PI,TWOPI,DEG,RAD,DAY,DTOKM
/PS /	SIG(8),TAU,TAUMAX
/PSANS1/	SSIG1,CSIG1
/PSANS2/	SUM2,SUM3,DIFF2,DIFF3,G,H,PSSQRT,PS,QS
/PSTIME/	CLO,FAKT,TOL

<u>Program data</u>: Size = 444_8 (292_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
CLO	1	DP	I	Initial value of σ_4
CSIG1	1	DP	O	$\cos \sigma_1$
DAY	1	DP	I	Value to convert days into hours, minutes, or seconds
FAKT	1	DP	I	FAKTPS from PSANS = A_4 (see appendix F)
ISET	1	DP	O	Flag to determine if the force model must be updated = 0 no (Force model still valid) = 1 yes (Outside range of validity for force model; update state vector to τma_x and reinitialize force model
QS	1	DP	I	q (sec. 4.2)
RI	1	DP	I	Inverse of the magnitude of the position vector of the satellite
SSIG1	1	DP	I	$\sin \sigma_1$

FORTRAN variable	Dimension	Type	Input/ output	Description
TAU	1	DP	0	New value of the independent variable (τ)
TAUMAX	1	DP	I	Range of validity for the force model
TFIN	1	DP	I	Final time desired for stopping the iteration
TIMEP	1	DP	I	Computed value of the physical time
TOL	1	DP	0	Allowable TOLerance between TFIN and TIME that will stop the iteration $$\|TFIN - TIME\| \le TOL$$
TWOPI	1	DP	I	2π
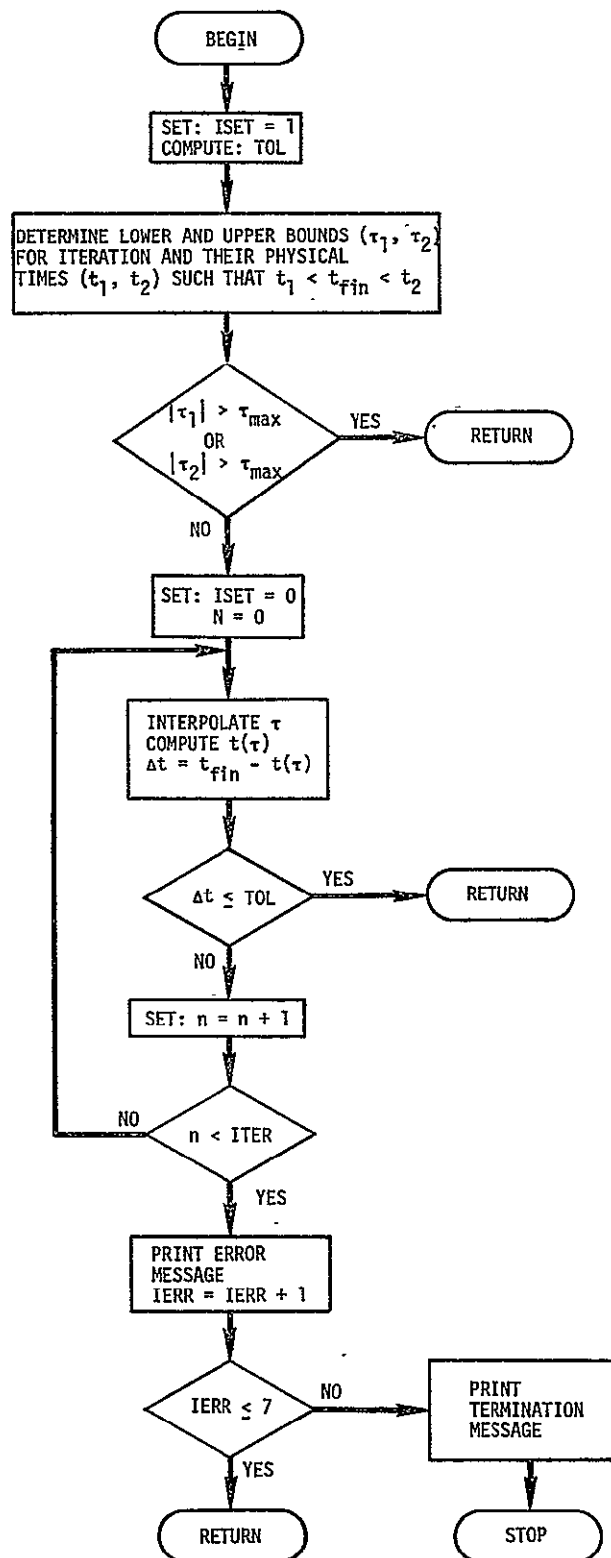
Figure 37.- TIMEPS flow chart.

3.3.32 TIMEXP (Subroutine)

Purpose: Compute coefficients of the expansion of the time equation

Calling sequence: CALL TIMEXP (XNU,ECC2,XMCAP,NMAX)

Called by: POTEXP

Subroutines/functions used: None

Named COMMON: None

Program data: Size = 200_8 (128_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
ECC2	1	DP	I	e^2
NMAX	1	I	I	Number of terms to compute
XMCAP	(7,18)	DP	0	Fourier coefficients of the time expansion
XNU	1	DP	I	Ratio of frequency of rotation of satellite to rotation of Earth

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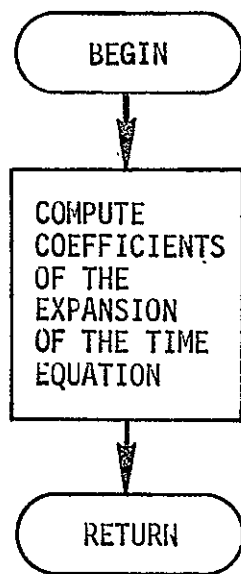
Figure 38.- TIMEXP flow chart.

3.3.33 XTOPS (Subroutine)

Purpose: Transform the Cartesian coordinates (X,V) into the PS (Poincaré-Similar) elements (σ,ρ)

Calling sequence: CALL XTOPS

Called by: ASOP, INPUT

Subroutines/functions used: GEOPOT

Named COMMON:
/CARTC /	X1,X2,X3,V1,V2,V3,TIME,ENERGY,R,RI
/CBODY /	XMU,XMUI,SQTMU,SQTMUI,EPS
/PS /	SIG(4),RHO(4),TAU
/PSANS1/	SSIG1,CSIG1,TWOL,XIQL,FAK
/PSANS2/	SUM2,SUM3,DIFF2,DIFF3,GC,HC,PSSQRT,PS,QS
/PSANS3/	QCAP
/PSTIME/	CLO,FAKTPS,TOL
/RETRO/	IRO
/RPOOL/	G1SQ,G2SQ,ESINPH,EROOT,EMINPH,SQTGHI,RCAP,RDOT, ZCAP1,ZCAP2,ECOSPH,XXX(4)

Equivalence: (HC,G3), (PHIC,RHO(1)), (XL,RHO(4))

Program data: Size = 576_8 (382_{10}) words compiled

FORTRAN variable	Dimension	Type	Input/ output	Description
(All equations are described in section 4.1)				
CLO	1	DP	O	Initial value of σ_4
CSIG1	1	DP	O	$\cos \sigma_1$
DIFF2	1	DP	O	$\rho_2{}^2 - \sigma_2{}^2$
DIFF3	1	DP	O	$\rho_3{}^2 - \sigma_3{}^2$
ENERGY	1	DP	O	Total orbital energy
FAK	1	DP	O	$(2L)^{-3/2}$
GC	1	DP	O	$G = \sqrt{G_x{}^2 + G_y{}^2 + G_z{}^2}$
HC	1	DP	O	$H = G_z$
IRO	1	I	O	Flag to determine if the orbit is retrograde. = -1 yes = 1 no

FORTRAN variable	Dimension	Type	Input/ output	Description
POT	1	DP	I	Magnitude of the Earth's gravitational potential
PS	1	DP	O	p
PSSQRT	1	DP	O	$\sqrt{p}$
QCAP	1	DP	O	Q
QS	1	DP	O	q
R	1	DP	O	Magnitude of the position vector of the satellite (r)
RHO	4	DP	O	$\rho_1 \rightarrow \rho_4$ in RHO(1) $\rightarrow$ RHO(4)
RI	1	DP	O	Inverse magnitude of the position vector of the satellite (1/r)
SIG	4	DP	O	$\sigma_1 \rightarrow \sigma_4$ in SIG(1) $\rightarrow$ SIG(4)
SSIG1	1	DP	O	$\sin \sigma_1$
SQTMUI	1	DP	I	$1/\sqrt{\mu}$
SUM2	1	DP	O	$\frac{1}{2}(\sigma_2^2 + \rho_2^2)$
SUM3	1	DP	O	$\frac{1}{2}(\sigma_3^2 + \rho_3^2)$
TAU	1	DP	O	PS independent variable; initially set to zero (rad)
TIME	1	DP	I	Initial physical time
TWOL	1	DP	O	$2L = 2\rho_4 = 2\sigma_8$
V1	1	DP	I	Components of the velocity vector
V2	1	DP	I	V1 = X component
V3	1	DP	I	V2 = Y component / V3 = Z component
XL	1	DP	O	ρ_4
X1	1	DP	I	Components of the position vector
X2	1	DP	I	X1 = X component
X3	1	DP	I	X2 = Y component / X3 = Z component

FORTRAN variable	Dimension	Type	Input/ output	Description
XIQL	1	DP	0	$\mu / \sqrt{2L}$
XMU	1	DP	I	μ
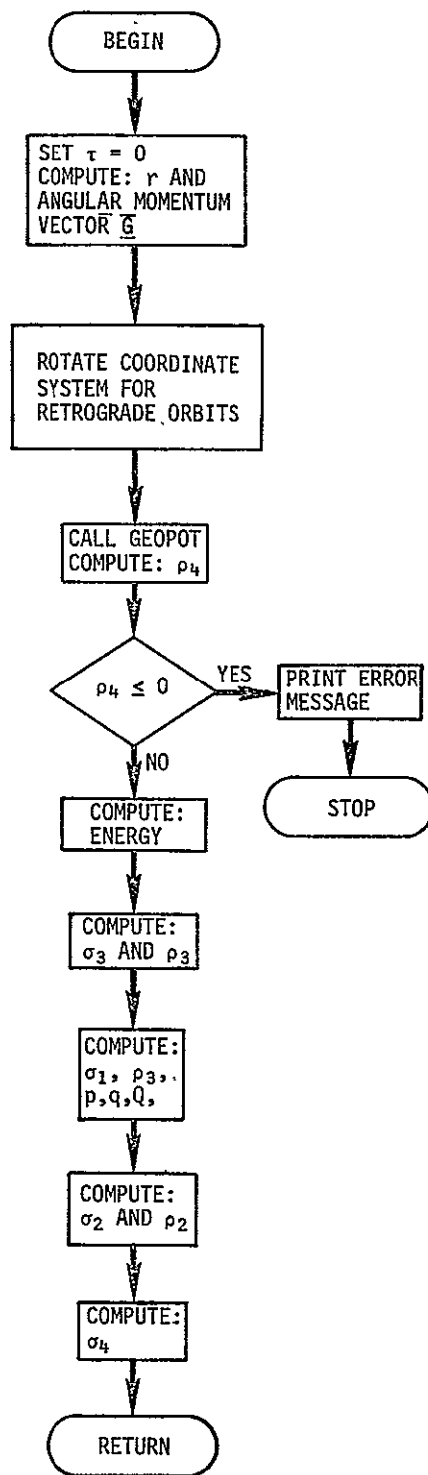
Figure 39.- XTOPS flow chart.

3.3.34 XYZAEI (Subroutine)

Purpose: Transform the Cartesian coordinates $(\vec{X}, \vec{V})$ into the Keplerian elements $(a, e, i, \omega, \Omega, M)$

Calling sequence: CALL XYZAEI

Called by: OUTPUT

Subroutines/functions used: None

Named COMMON:
/CARTC /	X1,X2,X3,V1,V2,V3,TIME,ENERGY,R,RI
/CBASIC/	PI,TWOPI,DEG,RAD,DAY,DTOKM
/CBODY /	XMU,XMUI,SQTMU,SQTMUI,EPS
/KEPLER/	A,E,XI,OMEGA,XNODE,XM
/RPOOL /	VSQ,RRDOT,ECOSE,ESINE,P,H,WX,WY,WZ,EA,RCOSF,RSINF,
	COSWF,SINWF,TEMP,RCOSL,RSINL,XXX(7)

Program data: Size = 406_8 (262_{10}) words compiled

Considers only elliptic motion

FORTRAN variable	Dimension	Type	Input/ output	Description
A	1	DP	O	Semimajor axis of the orbit
E	1	DP	O	Orbit eccentricity (e)
OMEGA	1	DP	O	Argument of pericenter (ω)
R	1	DP	O	Magnitude of the position vector of the satellite
SQTMUI	1	DP	I	$1/\sqrt{\mu}$
V1	1	DP	I	Components of the velocity vector
V2	1	DP	I	V1 = X component
V3	1	DP	I	V2 = Y component V3 = Z component
X1	1	DP	I	Components of the position vector
X2	1	DP	I	X1 = X component
X3	1	DP	I	X2 = Y component X3 = Z component
XM	1	DP	O	Mean anomaly (rad)

FORTRAN variable	Dimension	Type	Input/ output	Description
XMU	1	DP	I	Central body gravitational constant (μ)
XNODE	1	DP	0	Argument of the ascending node (Ω) (rad)
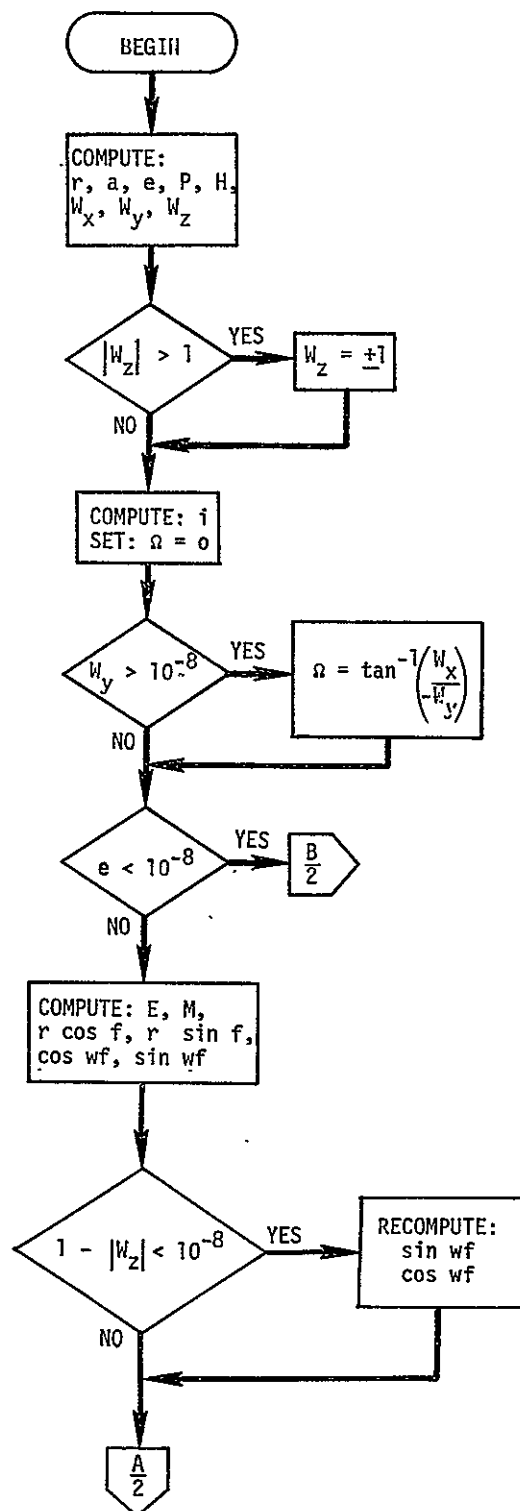
Figure 40.- XYZAEI flow chart.

A

$$\omega = \tan^{-1}\left(\frac{\sin \omega f}{\cos \omega f}\right)$$
$$-\tan^{-1}\left(\sin f/\cos f\right)$$

RETURN

B

SET: e = 0
        $\omega \neq 0$
COMPUTE:
r sin L, r cos L
$$M = \tan^{-1}\left(\frac{r \sin L}{r \cos L}\right)$$
        $-\Omega$

RETURN

Figure 40.- Concluded.

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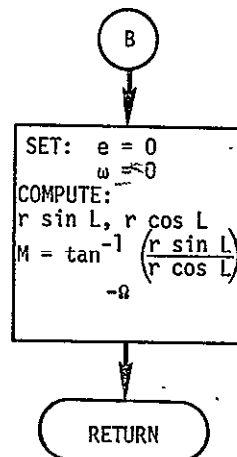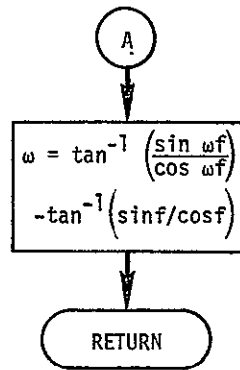
3.4 LABELED COMMON:

Notation: R - Real variable
           I - Integer variable   Type

           S - Single precision
           D - Double precision   Precision

/CARTC/; Cartesian coordinates of the satellite's position

Location	Name	Dimension	Type/precision	Description
1-3	X	3	R/D	Cartesian coordinates for the position of the satellite; X,Y,Z
4-6	V	3	R/D	Velocity vector of the satellite; V_x, V_y, V_z
7	TIME	1	R/D	Elapsed time (hr, min, or sec)
8	ENERGY	1	R/D	Total orbital energy
9	R	1	R/D	Magnitude of the position vector of the satellite
10	RI	1	R/D	Inverse magnitude of the position vector of the satellite

In subroutines: MAIN, AEIXYZ, ASOP, DENSTY, GEOPOT, INPUT,
                 OUTPUT, PREPD, PSTOX, TIMEPS, XTOPS, XYZAEI

/CBASIC/; Conversion constants

Location	Name	Dimension	Type/precision	Description
1	PI	1	R/D	π
2	TWOPI	1	R/D	2π
3	DEG	1	R/D	$\pi/180$
4	RAD	1	R/D	$180/\pi$

Location	Name	Dimension	Type/precision	Description
5	DAY	1	R/D	Converts days into hours, minutes, or seconds
6	DTOKM	1	R/D	Converts distance into kilometers

In subroutines: MAIN, ASOP, CONST, DENSTY, GEOPOT, INITAL, INPUT, OUTPUT, POTEXP, PREPD, PREPS, PSANS, SUN, TIMEPS, XYZAEI

/CBODY/; Gravitational variables

Location	Name	Dimension	Tuype/precision	Description
1	XMU	1	R/D	Gravitational constant of the central body (μ)
2	XMUI	1	R/D	$1/\mu$
3	SQTMU	1	R/D	$\sqrt{\mu}$
4	SQTMUI	1	R/D	$\sqrt{1/\mu}$
5	EPS	1	R/D	$\varepsilon = 3/2 \ (\mu \ J_2 \ R_e^2)$ where $J_2 = J_2$ geopotential coefficient R_e = Equatorial radius of the central body

In subroutines: AEIXYZ, CANFOR, CONST, GEOPOT, LONGPP, POTEXP, PREPD, PSANS, PSTOX, XTOPS, XYZAEI

/CONSTW/

Location	Name	Dimension	Type/precision	Description
1	TWO3	1	R/D	2/3
2	BY3	1	R/D	1/3
3	BY6	1	R/D	1/6
4	CN	1	R/D	± 1 depending on NN = +1 initializing (NN=0) = -1 computing (NN≠0)

In subroutines: LONGPP, PSANS

/CPRINT/

Location	Name	Dimension	Type/precision	Description
1	PRINT	1	R/D	Input parameters, see section 2.1.1; Note: IPRINT is set to 3 if re-entry condition exists
2	IPRINT	1	I	
3	IPSPRT	1	I	
4	IUNITS	1	I	

In subroutines: MAIN, CONST, INPUT, OUTPUT

/DATMOS/; Atmospheric parameters

Location	Name	Dimension	Type/precision	Description
1	FBAR	1	R/D	Averaged value for the solar flux coefficient, $\overline{F}_{10.7}$
2	XKP	1	R/D	Averaged value for the geomagnetic index, K_p
3	SLDAY	1	R/D	Magnitude of the seasonal latitudinal density variation
4	SADAY	1	R/D	Magnitude of the semiannual density variation

In subroutines: DENSTY, INITAL

/DATMO1/; Atmospheric parameters

Location	Name	Dimension	Type/ precision	Description
1	SRAB	1	R/D	Sine of bulge right ascension
2	CRAB	1	R/D	Cosine of bulge right ascension
3	SDEC	1	R/D	Sine of bulge declination
4	CDEC	1	R/D	Cosine of bulge declination
5	RR	1	R/D	Magnitude of the diurnal change in the exospheric temperature
6	TC	1	R/D	Nighttime minimum of the global exospheric temperature (^{O}K)
7	TG	1	R/D	Variation in the exospheric temperature due to geomagnetic activity (^{O}K)

In subroutines: DENSTY, INITAL

/DBETAS/; Powers of beta

Location	Name	Dimension	Type/ precision	Description
1	BETA1	1	R/D	β
2	BETA2	1	R/D	β^2
3	BETA3	1	R/D	β^3
4	BETA4	1	R/D	β^4

In subroutines: CANFOR, PREPD, TABLE

/DCOEFF/; Coefficients for the Jacchia 71/Lineberry atmospheric density model

Location	Name	Dimension	Type/ precision	Description
1-27	A	(3,3,3)	R/D	Parameters for determining the base altitude

Location	Name	Dimension	Type/precision	Description
28-54	B	(3,9)	R/D	Parameters for determining the T_∞ density profile
55-81	C	(3,9)	R/D	Parameters for computing the annual variation
82-93	D	(3,4)	R/D	Parameters for computing the seasonal latitudinal variation

In subroutines: DENSTY, INITAL

/DENS/; ASOP atmospheric density model parameters

Location	Name	Dimension	Type/precision	Description
1-10	B	10	R/D	Coefficients of the atmospheric density model[a]
11	DS1	1	R/D	Coefficients of the Fourier series describing the diurnal bulge[a]
12	DS2	1	R/D	
13	DC1	1	R/D	
14	DC2	1	R/D	
15	OO	1	R/D	Coefficients of the Fourier series describing the density variation due to short period J_2 changes in height[a]
16	OC2	1	R/D	
17	OS2	1	R/D	
18	OC1	1	R/D	
19	OS1	1	R/D	

In subroutines: LONGPP, PREPD, PSANS, TABLE

/DETE/; First-order long period parameters

Location	Name	Dimension	Type/precision	Description
1	SHAT	1	R/D	First-order, long-period generating function [b]
2	SHATP	1	R/D	Derivatives of the first-order, long-period generating function with respect to p,e^2,b,χ_1 and ψ_1 [b]
3	SHATE2	1	R/D	
4	SHATB	1	R/D	
5	SHATXI	1	R/D	
6	SHATPI	1	R/D	

In subroutines: DETERM,LONGPP

/DRAG/; Input parameters

Location	Name	Dimension	Type/precision	Description
1	CD	1	R/D	Input parameters, see section 2.1.1
2	AREA	1	R/D	
3	XMASS	1	R/D	
4	CDRAG	1	R/D	

In subroutines: CONST, INPUT, PREPD

/DRAG1/; Drag functions

Location	Name	Dimension	Type/precision	Description
1-8	CFORCE	8	R/D	Drag force defined in PS elements
9	T4	1	R/D	Magnitude of the quadratic variation in the mean anomaly

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[a]Reference 7, pp. 18-22.
[b]Reference 8.

Location	Name	Dimension	Type/precision	Description
10	TLINER	1	R/D	Linear change in time due to drag
11	TEMPO	1	R/D	Second order correction for density due to drag

In subroutines: DRAG, PREPD

/DTABLE/; Averaged drag functions

Location	Name	Dimension	Type/precision	Description
1-12	TT	12	R/D	Table of averaged Fourier series in σ_1

In subroutines: CANFOR, TABLE

/ECC/; Eccentricity parameters

Location	Name	Dimension	Type/precision	Description
1	ES	1	R/D	e
2	ESSQ	1	R/D	e^2

In subroutines: DETERM, FPRIME, LONGPP

/END/; Input stopping parameters

Location	Name	Dimension	Type/precision	Description
1	STOP	1	R/D	Input parameters, see section 2.1.1; Note: ISTOP is reset to 3 if reentry condition exists
2	ISTOP	1	I	

In subroutines: MAIN, CONST, INPUT, OUTPUT, PREPD

/EPOCH/; Input dating parameters

Location	Name	Dimension	Type/precision	Description
1-6	DATE	6	R/D	Input parameter, see section 2.1.1
7	XJDATE	1	R/D	Julian date corresponding to DATE

In subroutines: CONST, INITAL, INPUT, OUTPUT, PREPS

/EXPCOF/; Binomial coefficients and Fourier coefficients of the powers of cosine and sine

Location	Name	Dimension	Type/precision	Description
1-200	A	200	R/D	Array containing the binomial coefficients
201-400	B	200	R/D	Array containing the Fourier coefficients for cosine and sine raised to a power
401	NDEX0	1	I	Zero Index to the NDEX array
402-419	NDEX	18	I	Array of pointers to the A and B coefficients
420	IEXPFL	1	I	Flag to determine if the A and B arrays have been computed = 0 no = 1 yes

In subroutines: COEFF, DETERM, FPRIME, POTEXP

/FORSAV/; Fourier series expansion coefficients

Location	Name	Dimension	Type/precision	Description
1-6	Z	6	R/D	χ^i_o
7-60	ZC	(9,6)	R/D	χ^i_j
61-114	ZS	(9,6)	R/D	ψ^i_j
115-120	DZ	6	R/D	$\hat{\chi}^i_o$
121-174	DZC	(9,6)	R/D	$\hat{\chi}^i_j$
175-258	DZS	(9,6)	R/D	$\hat{\psi}^i_j$

Coefficients of the Fourier series expansion of the disturbing function in PS elements[a]

In subroutines: PREPD, TABLE

/FP/; Second-order zonal parameters

Location	Name	Dimension	Type/precision	Description
1	FHAT	1	R/D	Second-order zonal Hamiltonian[b]
2	FHATP	1	R/D	Derivatives of the second-order zonal Hamiltonian with respect to p, e^2 and b [b]
3	FHATE^2	1	R/D	
4	FHATB	1	R/D	

In subroutines: FPRIME, LONGPP

/GEO/; Geopotential coefficients for the Earth

Location	Name	Dimension	Type/precision	Description
1	RE	1	R/D	Equatorial radius of the central body (Earth)
2	CJ2	1	R/D	J_2 geopotential coefficient of the central body (Earth)

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[a]Reference 7, pp. 43-45.
[b]Reference 8.

Location	Name	Dimension	Type/ precision	Description
3-189	CS	187	R/D	C coefficients of the geopotential model in the unnormalized form
190-376	SS	187	R/D	S coefficients of the geopotential model in the unnormalized form
377	IGEOFL	1	R/D	Flag to determine if C and S arrays are set = 0 no = 1 .yes

In subroutines: ASOP, CONST, DETERM, FPRIME, GEOPOT, POTEXP, PREPD, PREPT

/GMTROT/; Greenwich Meridian rotational parameters referenced to·a desired epoch

Location	Name	Dimension	Type/ precision	Description
1	WE	1	R/D	Rotational rate of the Earth
2	THETAO	1	R/D	Initial hour angle of the Earth

In subroutines: CANFOR, CONST, GEOPOT, POTEXP, PREPD

/HAMDS/; Hamiltonian derivatives

Location	Name	Dimension	Type/ precision	Description
1-4	DSF	4	R/D	$\partial f/\partial \beta_K$ k=1, 2, 3, 4
5-8	DSB	4	R/D	$\partial b/\partial \beta_K$· k=1, 2, 3, 4

In subroutines: LONGPP, PSANS

/KEPLER/; Keplerian elements of the satellite's orbit

Location	Name	Dimension	Type/precision	Description
1	A	1	R/D	Semimajor axis of the orbit (a)
2	E	1	R/D	Eccentricity (e)
3	XI	1	R/D	Orbital inclination to the Equator (i)
4	OMEGA	1	R/D	Argument of pericenter (ω)
5	XNODE	1	R/D	Argument of the ascending (Ω) node
6	XM	1	R/D	Mean anomaly (M)

In subroutines: AEIXYZ, INPUT, OUTPUT, XYZAEI

/PERTRB/; Input perturbation flags

Location	Name	Dimension	Type/precision	Description
1	IDRAG	1	I	Input parameters, see section 2.1.1
2	ILONG	1	I	

In subroutines: ASOP, CONST, GEOPOT, INPUT, PREPD, PSANS

/PS/; PS elements and independent variable

Location	Name	Dimension	Type/precision	Description
1-4	SIG	4	R/D	PS elements σ_1, σ_2, σ_3, σ_4
5-8	RHO	4	R/D	PS elements ρ_1, ρ_2, ρ_3, ρ_4
9	TAU	1	R/D	Independent variable of the PS elements (τ)
10	TAUMAX	1	R/D	Range of validity for the force model; When τ exceeds this value (τ_{max}), the force model must be reinitialized

Location	Name	Dimension	Type/precision	Description
11	TAUINT	1	R/D	Initial value of τ for which the force model is valid; initially set to 0

In subroutines: MAIN, ASOP, CANFOR, DRAG, LONGPP, OUTPUT, POTEXP, PREPD, PSANS, PSTOX, TABLE, TIMEPS, XTOPS

/PSANSV/; PS formulation variables

Location	Name	Dimension	Type/precision	Description
1-4	FACTOR	4	R/D	Derivatives of the DS Hamiltonian and its combinations (A_1, A_2, A_3 and A_4 in appendix F)
5-12	SIGINI	8	R/D	Initial values of the σ's and ρ's

In subroutines: LONGPP, PREPD, PSANS

/PSANS1/; PS formulation variables

Location	Name	Dimension	Type/precision	Description
1	SSIG1	1	D/P	$\sin \sigma_1$
2	CSIG1	1	D/P	$\cos \sigma_1$
3	TWOL	1	D/P	$2L$
4	XIQL	1	D/P	$\mu / \sqrt{2L}$
5	FAK	1	D/P	$(2L)^{-3/2}$

In subroutines: CANFOR, LONGPP, POTEXP, PREPD, PSANS, PSTOX, XTOPS

/PSANS2/; PS formulation variables

Location	Name	Dimension	Type/precision	Description
1	SUM2	1	D/P	$(\sigma_2^2 + \rho_2^2)/2$
2	SUM3	1	D/P	$(\sigma_3^2 + \rho_3^2)/2$
3	DIFF2	1	D/P	$\rho_2^2 - \sigma_2^2$
4	DIFF3	1	D/P	$\rho_3^2 - \sigma_3^2$
5	G	1	D/P	G
6	H	1	D/P	H
7	PSSQRT	1	D/P	$\sqrt{p}$
8	PS	1	D/P	p
9	QS	1	D/P	q

In subroutines: CANFOR, DETERM, FPRIME, LONGPP, POTEXP, PREPD, PSANS, PSTOX, TIMEPS, XTOPS

/PSANS3/; PS formulation variables

Location	Name	Dimension	Type/precision	Description
1	QC	1	R/D	Q
2	EROOT	1	R/D	$\sqrt{2L\,p/u}$
3	X3ROOT	1	R/D	$\left(\sqrt{4G - \sigma_3^2 - \rho_3^2}\right)/G$

In subroutines: LONGPP, POTEXP, PREPD, PSANS, PSTOX, XTOPS

/PSTIME/; PS parameters used for stopping on a specific final time.

Location	Name	Dimension	Type/precision	Description
1	CLO	1	R/D	Initial value of σ_4 (set when initializing, NEWX = 0)
2	FAKTPS	1	R/D	$\partial\sigma_4/\partial\tau$
3	TOL	1	R/D	Tolerance criteria for the iteration stopping procedure

In subroutines: MAIN, PSANS, PSTOX, TIMEPS, XTOPS

/RETRO/; Retrograde parameter

Location	Name	Dimension	Type/precision	Description
1	IRO	1	I	Flag to determine retrograde orbit = -1 yes, it is a retrograde orbit = 1 no, it is a retrograde orbit

In subroutines: CANFOR, DETERM, POTEXP, PREPD, PSTOX, XTOPS

/RPOOL/,/RPOOLA/; Temporary variables

Location	Name	Dimension	Type/precision	Description
Variable	--	1393	R/D	Temporary real variables; this COMMON block is used to help save storage within the ASOP program

In subroutines: AEIXYZ, DETERM, FPRIME, GEOPOT, LONGPP, POTEXP, PREPT, PSTOX, XTOPS, XYZAEI

/SUNPAR/; Orbit parameters of the Sun

Location	Name	Dimension	Type/ precision	Description
1	XNS	1	R/D	Mean motion of the Sun
2	XLS0	1	R/D	Mean anomaly of the Sun
3	A	1	R/D	Semimajor axis of the Sun
4	E	1	R/D	Eccentricity of the Sun's orbit
5	AEROOT	1	R/D	Argument of perigee of the Sun
6-7	B1	2	R/D	Coefficients to transform the position of the Sun from the
8-9	B2	2	R/D	orbital plane to the mean-of-epoch equatorial reference
10-11	B3	2	R/D	system

In subroutines: PREPD, PREPS, SUN

/SUNPOS/; Sun's position referenced to an input epoch

Location	Name	Dimension	Type/ precision	Description
1	XS	1	R/D	Position vector of the Sun in Earth's inertial equato-rial system (XS, YS, ZS)
2	YS	1	R/D	
3	ZS	1	R/D	
4	RS	1	·R/D	Magnitude of the position vector of the Sun
5	RAS	1	R/D	Right ascension of the Sun
6	DECS	1	R/D	Declination of the Sun

In subroutines: INITAL, PREPD, SUN

/S1STAD/

Location	Name	Dimension	Type/ precision	Description
1-8	GC	8	R/D	$\partial G/\partial\sigma_k$, $\partial G/\partial\rho_k$ k=1,2,3,4
9-16	P	8	R/D	$\partial p/\partial\sigma_k$, $\partial p/\partial\rho_k$ k=1,2,3,4
17-24	Q	8	R/D	$\partial q/\partial\sigma_k$, $\partial q/\partial\rho_k$ k=1,2,3,4
25-32	HC	8	R/D	$\partial H/\partial\sigma_k$, $\partial H/\partial\rho_k$ k=1,2,3,4
33-40	QCV	8	R/D	$\partial Q/\partial\sigma_k$, $\partial Q/\partial\sigma_k$ k=1,2,3,4

In subroutines: LONGPP, PSANS

/S1STAV/

Location	Name	Dimension	Type/ precision	Description
1	GIN	1	R/D	G^{-1}
2	HOG	1	R/D	H/G
3	GPH	1	R/D	G+H
4	BS	1	R/D	b = 1 - H/G

Location	Name	Dimension	Type/ precision	Description
5	FS	1	R/D	f = 1/pq
6	GINSQ	1	R/D	G^{-2}

In subroutines: DETERM, FPRIME, LONGPP, PSANS

/TESS/; Input parameters for the geopotential model (zonal and tesseral terms)

Location	Name	Dimension	Type/ precision	Description
1	NMAX	1	I	Input parameters; see section 2.1.1
2	MMAX	1	I	

In subroutines: GEOPOT, INPUT, POTEXP

/XIPSI/

Location	Name	Dimension	Type/ precision	Description
1	XI1	1	R/D	χ_1 = e sin I cos g I = inclination g = argument of perigee
2	PSI1		R/D	ψ_1 = e sin I sin g I = inclination g = argument of perigee

In subroutines; DETERM, LONGPP

4.0 PS ELEMENT FORMULATION

The PS (Poincare-Similar) element formulation is described in the following subsections. An exact development of this element set is described in reference 5, and a description of the variables can be found in Appendix E.

4.1 TRANSFORMATION FROM CARTESIAN COORDINATES TO PS ELEMENTS (XTOPS)

The transformation into the PS element set (σ, ρ) from a given set of Cartesian coordinates (X, V, r) is accomplished within the subroutine XTOPS. It is assumed that these coordinates are the initial conditions so that the physical time t and the independent variable τ both equal zero. The PS elements are then computed using the equations

$$\sigma_1 = \tan^{-1}\left(\frac{X_x + R^*\sigma_3}{X_y + R^*\rho_3}\right) \qquad \rho_1 = G - \sqrt{G^2 + 2r^2 V} + \frac{\mu}{\sqrt{2\rho_4}}$$

$$\sigma_2 = Z_2 \cos \sigma_1 - Z_1 \sin \sigma_1 \qquad \rho_2 = Z_2 \sin \sigma_1 + Z_1 \cos \sigma_1$$

$$\sigma_3 = -2G_x/\sqrt{2(G + G_z)} \qquad \rho_3 = 2G_y/\sqrt{2(G + G_z)}$$

$$\sigma_4 = t - \frac{\mu}{(2\rho_4)^{3/2}}(E - \phi) \qquad \rho_4 = \frac{\mu}{r} - \frac{1}{2}\vec{V}^2 - V, \text{ where } V \text{ is the perturbing potential}$$

$$- \frac{r}{p} Z_2 Q \sqrt{1 - e^2}$$

The other required relations are:

$$G_x = X_y V_z - X_z V_y \quad , \quad G_y = X_z V_x - X_x V_z \quad , \quad G_z = X_x V_y - X_y V_x$$

$$G = \sqrt{G_x^2 + G_y^2 + G_z^2}$$

$$p = 1/\mu \left(G - \rho_1 + \mu/\sqrt{2\rho_4}\right)^2 \quad , \quad q = G - 1/2 \rho_1 + \mu/2\sqrt{2\rho_4}$$

$$R^* = rR/2G \quad , \quad R = 2GX_z/r\sqrt{2(G + G_z)}$$

$$Z_1 = (p/r - 1)/Q \quad , \quad Z_2 = \dot{r}p/\left[Q(2q - G)\right]$$

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$$Q = 1/\mu \left[\rho_4 \left(2\mu/\sqrt{2\rho_4} + G_z - G \right) \right]^{1/2}$$

$$\dot{r} = \frac{X \cdot V}{r} \quad , \quad \sqrt{1 - e^2} = p \sqrt{2\rho_4/\mu}$$

$$E - \phi = -2 \tan^{-1}\left[Z_2 Q/(1 + \sqrt{1 - e^2} + Z_1 Q) \right]$$

This transformation is performed only once for a given set of Cartesian coordinates unless the desired value of τ is greater than the maximum value for which the force model is still valid. This transformation is repeated whenever the force model is reinitialized.

4.2 TRANSFORMATION FROM PS ELEMENTS TO CARTESIAN COORDINATES (PSTOX)

This transformation is performed when any intermediate printout is desired or when the final condition is met. Therefore, this transformation is coded with emphasis on the speed of calculation.

The Cartesian coordinates defined in terms of the PS elements are given by the equations:

$$X_x = r \cos \sigma_1 - R^* \sigma_3 \quad , \quad V_x = \dot{r} \cos \sigma_1 - \frac{G}{r} \sin \sigma_1 - \dot{R}^* \sigma_3$$

$$X_y = r \sin \sigma_1 - R^* \rho_3 \quad , \quad V_y = \dot{r} \sin \sigma_1 - \frac{G}{r} \cos \sigma_1 - \dot{R}^* \rho_3$$

$$X_z = R^* \sqrt{2(G + H)} \quad , \quad V_z = \dot{R}^* \sqrt{2(G + H)}$$

The other necessary relationships are

$$r = p/(1 + e \cos \phi) \quad , \quad R^* = rR/2G$$

$$\dot{r} = e \sin \phi/p \left[2q - \rho_1 + 1/2 \left(\rho_2^2 + \sigma_2^2 \right) \right]$$

$$\dot{R}^* = (R\dot{r} + \dot{R}r)/2G \quad , \quad \dot{R} = \frac{G}{r^2}(\rho_3 \cos \sigma_1 - \sigma_3 \sin \sigma_1)$$

$$R = \rho_3 \sin \sigma_1 + \sigma_3 \cos \sigma_1 \quad , \quad G = \rho_1 - 1/2 \left(\rho_2^2 + \sigma_2^2 \right)$$

$$H = G - 1/2 \ (\rho_3{}^2 + \sigma_3{}^2) \quad , \quad q = -1/2 \ (\rho_2{}^2 + \sigma_2{}^2 - \rho_1 - \mu/\sqrt{2\rho_4})$$

$$p = 1/\mu \ \left[\mu/\sqrt{2\rho_4} - 1/2 \ (\rho_2{}^2 + \sigma_2{}^2) \right]$$

$$e \sin \phi = Q(\rho_2 \sin \sigma_1 + \sigma_2 \cos \sigma_1)$$

$$e \cos \phi = Q(\rho_2 \cos \sigma_1 - \sigma_2 \sin \sigma_1)$$

$$Q = 1/\mu \ \left\{ \rho_4 \left[2\mu/\sqrt{2\rho_4} - 1/2 \ (\rho_2{}^2 + \sigma_2{}^2) \right] \right\}^{1/2}$$

The physical time t is computed using

$$t = \sigma_4 + \frac{\mu}{(2\rho_4)^{3/2}} \left(E - \phi - \frac{r}{p} \ e \sin \phi \ \sqrt{1 - e^2} \right) \tag{4.2a}$$

where the expression for $E - \phi$ is given by

$$E - \phi = -2 \tan^{-1} \left[e \sin \phi / (1 + \sqrt{1 - e^2} + e \cos \phi) \right]$$

4.3 TIME TERMINATION PROCEDURE

Because the PS element set uses the true anomaly τ as the independent variable, an iteration procedure is necessary to stop at a specific time t_{final}. Within the ASOP program, this iteration is performed by the TIMEPS subroutine in the following manner:

An expression for the derivative of time with respect to the true anomaly τ is given in the PS theory as

$$\frac{dt}{d\tau} = r^2/q$$

This expression can be linearly approximated by

$$\frac{\Delta t}{\Delta \tau} = r^2/q \tag{4.3a}$$

where $\Delta t = t_{final} - t_n$ and $\Delta \tau = \tau_{n+1} - \tau_n$. Equation 4.3a then yields a recursive formula for refining an initial estimate of τ of

$$\tau_{n+1} = \tau_n - \Delta t \ q/r^2 \tag{4.3b}$$

Using equation 4.3b, an initial estimate of τ is refined until the associated value of t equals the desired final time t_{final}.

In order to start the iteration, a suitable initial value of τ is necessary. This value is determined by first assuming that a circular orbit is being used. With this assumption, equation 4.2a reduces to

$$\sigma_4 = t$$

and an initial approximation for τ can be written as

$$\tau_0 = (t_{final} - \sigma_4)/\frac{\partial \sigma_4}{\partial \tau}$$

The value of $\partial \sigma_4/\partial \tau$ is taken from the analytical theory (ref. 5) during the initialization procedure.

Therefore, the full algorithm is

a. Set the iteration counter n to zero, and compute the initial approximation for $\tau_n = \tau_0$.

b. Determine the PS elements at τ_n.

c. Determine the time t_n at τ_n.

d. If $|t_{final} - t_n| \leq$ TOLerance, then STOP; otherwise $n = n + 1$

e. If $n > n_{max}$, then print a diagnostic message and STOP

f. Compute a new approximation for τ_n using

$$\tau_n = \tau_{n-1} - (t_{n-1} - t_{final})q/r^2$$

g. Go to step b.

Values for TOLerance and n_{max} have been preset within the TIMEPS subroutine to 10^{-7} and 15, respectively.

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APPENDIX A

AVAILABLE UNITS AND PHYSICAL CONSTANTS

Within the ASOP program, there are a number of options for the input units. The compatibility of the input values with the selected physical constants is the responsibility of the user and is controlled by the input flag IUNITS. The following constants are listed in order according to the value assigned to IUNITS. If no value for IUNITS is explicitly given, then 1 is assumed.

R_e = Earth radius (equatorial)

= 6378.140 km   (IUNITS = 1,5)

= 3443.920 nm   (IUNITS = 2,6)

= 2.092566×10^7 ft   (IUNITS = 3)

= 6.378140×10^6 m   (IUNITS = 4)

= 1.0 E.r.   (IUNITS = 7)

μ = Gravitational constant of the Earth

= 3.986013×10^5 km^3/sec^2 (IUNITS = 1)

= 6.275029×10^4 nm^3/sec^2 (IUNITS = 2)

= 1.407647×10^{16} ft^3/sec^2 (IUNITS = 3)

= 3.986013×10^{14} m^3/sec^2 (IUNITS = 4)

= 5.165873×10^{12} km^3/hr^2 (IUNITS = 5)

= 8.132438×10^{11} nm^3/hr^2 (IUNITS = 6)

= 5.530432×10^{-3} E.r.^{3}/min^2 (IUNITS = 7)

DAY = Time conversion

= 8.64×10^4 sec/day   (IUNITS = 1,2,3,4)

= 1.44×10^3 min/day   (IUNITS = 7)

= 2.40×10^1 hr/day   (IUNITS = 5,6)

$J_2 = J_2$ coefficient of the Earth's geopotential

$= 1.082637 \times 10^{-3}$

$\epsilon = EPS = 3/2 \ (\mu \ J_2 \ R_e^2)$

$\pi = PI = 3.14159 \ 26535 \ 898$

If Keplerian elements are input (IEL = 1), then the semimajor axis must be input in a distance compatible with the selected value of IUNITS, i.e., if IUNITS = 2, then EL(1) must be given in nautical miles; if IUNITS = 4, then EL(1) must be in meters, etc. All angles must be given in degrees.

For Cartesian coordinates, the input unit must be

IUNITS	EL(1) → EL(3)	EL(4) → EL(6)
1	km	km/sec
2	nm	nm/sec
3	ft	ft/sec
4	m	m/sec
5	km	km/hr
6	nm	nm/hr
7	E.r.	E.r./min

All computations within ASOP are done using the input units.

APPENDIX B

REQUIRED CONTROL CARDS

1. >@QUAL FM6-N08569

2. >@ASG,A *NUMEG.

3. >@XQT *NUMEG.ASOP-PROG

4. INPUT DATA USING NAMELIST '$INPUT'

5. >ƀ$INPUT   >@ADD filename.element name

    Necessary input parameters;   or   > Input the necessary correction
    see section 2.1.1. All   if needed.
    parameters must be preceded
    by at least one space.

6. >ƀ$END   >ƀ$END

    Initial output

7. ENTER: X = EXECUTE; S = STOP; C = CHANGE INPUT

    >

    If an X is entered, the program will printout the desired information,
    and the sequence will begin again at line 4.

    If an S is entered in response to the prompt '>', then the message
    NORMAL PROGRAM TERMINATION should appear.

    If a C is entered, the program will respond with **CHANGE DATA USING
    THE NAMELIST '$INPUT'** and the sequence will begin again at line 5.

8. >@EOF
    NORMAL PROGRAM TERMINATION

    > = system prompt;
    ƀ = necessary space

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APPENDIX C

ASOP DEFAULT VALUES

IEL   = 1 (Keplerian elements)

EL   = None; must be input by user

STOP   = 100.0D0

ISTOP   = 2 (STOP value is in revolutions)

PRINT   = 0.0D0.

IPRINT = 0 (No printout; PRINT value ignored)

IDRAG   = 1 (Drag terms desired)

AREA   = 185.3D0 (m^2)

CD   = 2.2D0   Shuttle average

XMASS   = 90700.0D0 (kg)

ILONG   = 1 (J_2 and short period secular terms)

NMAX   = 2 (Include $J_{2,0}$ zonal term)

MMAX   = 0 (No tesseral terms)

IPSPRT = 0 (Do not print PS elements)

IUNITS = 1 (Input and output values are given as

        distance = km
        velocity = km/sec
        time   = day
        angles   = deg)

APPENDIX D

SUBROUTINE STORAGE REQUIREMENTS

FORTRAN compiler: SOE3

MAP processor   : 28R2 RL71-3

Subroutine name	Storage requirements, words	
	Octal	Decimal
MAIN	171	121
AEIXYZ	302	194
ASOP[a]	235	46
CANFOR[a]	524	340
CDTOJD	176	126
COEFF[a]	322	210
CONST[a]	406	262
DENSTY[a,b]	672	442
DETERM[a]	1 467	823
DRAG[a]	115	77
FPRIME[a]	1 177	639
GEOPOT[a]	430	280
ILOG10[a]	106	70
INITAL[a,b]	255	173
INPUT	407	263
JDTOCD	160	112

---

[a]ASOP subroutine package programs.
[b]These subroutines are models that may or may not be found in user's own library.

Subroutine name	Storage requirements, words	
	Octal	Decimal
LONGPP[a]	1 705	965
MATIN[a,b]	573	379
MTOECC[a]	163	115
OUTPUT	710	456
POTEXP[a]	2 214	1 164
PREPD[a]	2 212	1 162
PREPS[a,b]	276	190
PREPT[a,b]	1 031	537
PSANS[a]	1 712	970
PSTOX[a]	305	197
RECUR[a]	163	115
SACT[a,b]	545	357
SUN[a,b]	153	107
TABLE[a]	1 423	787
TIMEPS[a]	502	322
TIMEXP[a]	200	128
XTOPS[a]	576	382
XYZAEI	406	262

Subtotals

ASOP program	30 745	12 773
ASOP subroutine package	25 747	11 239
COMMON block storage requirements	13 067	5 687

---

[a]ASOP subroutine package programs.
[b]These subroutines are models that may or may not be found in user's own library.

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Totals	Octal	Decimal
ASOP program	44 034	18 460
ASOP. subroutine package	41 036	16 926

.

The subroutines labeled with a 'c' are models that may or may not be found in the user's own subroutine library. They also may be subject to change depending on the user's needs or requirements. For instance, PREPT initializes the coefficients of a very accurate but extremely large geopotential model (18th order x 18th degree). If such accuracy is not required a 4 x 4 model, for example, may be included to reduce storage requirements. The storage requirements for these standard library routines are 4211_8 or 2185_{10} words. Therefore, the storage requirements without these standard library routine options are:

	Octal	Decimal
Totals		
ASOP program	37 623	16 275
ASOP subroutine package	34 625	14 741

Some common blocks may also be reduced if, for instance, a smaller geopotential model was used. This would further reduce the storage requirements.

Values given for the storage requirements of the individual subroutines are values returned by the FORTRAN compiler when forming a relocatable element. The final, executable program will require more space because of the system library modules that must also be included.

The ASOP program, with system routines and load tables, occupies over $28\ 000_{10}$ words of storage. This has been reduced to approximately $21\ 500_{10}$ words by a simple overlay structure. Most of the program remains available at all times. Only three sections overlap each other: (1) input subroutines, (2) initialization of the drag model, and (3) computation of mean energy due to tesseral and sectorial geopotential harmonics. This overlay structure does not increase execution time significantly because the initialization routines are overlayed with updating routines. The core is swapped out only once unless more input is brought in.[a]

---

[a]For long prediction intervals (STOP 3 days) or for conditions near reentry, the initialization is performed more than once to account for second order perturbations.

Two test cases were run to compare the total CPU time needed to initialize and update an orbit prediction. Both were orbit predictions for 1 day with stop options on time and revolutions. The first test case consists of four sets of conditions:

a. An 8th order 8th degree (8 x 8) geopotential model and a diurnal atmospheric drag model

b. An 8 x 0 (no tesseral) geopotential model and a diurnal atmospheric drag model

c. A 5 x 2 geopotential model and a static atmospheric drag model

d. An 8 x 0 geopotential model with no drag

The second test case consists of three sets of conditions. There is no drag because of the high eccentricity.

a. An 8 x 8 geopotential model

b. An 8 x 0 (no tesseral) geopotential model

c. A 5 x 2 geopotential model

Initialization is the time spent in accepting input data, transforming the state, and preparing to update the state. Updating is the process of actually propagating the state to a desired condition. Thus, if a particular problem requires 50 ms to initialize and 8 ms to update, then the total execution time is $50 + 8 = 58$ ms. If nine intermediate states are desired, then the total execution time required to determine the nine intermediate states and one final state is $50 + 9 * 8 + 8 = 130$ ms. Note that the initialization usually requires more time than this to update, but it is required only once [a].

---

[a]For long prediction intervals (STOP > 3 days) or for conditions near reentry, the initialization is performed more than once to account for second-order perturbations.

Test Case 1: Small eccentricity (0.02) orbit

Initial conditions: A = 6712.39 km   e = .02   i = 30°
                     W = 30°   Ω = 20°   M = 20°

			Stop on time[1]	Stop on revs[1]
a.	8 x 8	Initialize	350 ms	350 ms
		Update	17 ms	8 ms
b.	8 x 0	Initialize	50 ms	50 ms
		Update	17 ms	8 ms
c.	5 x 2	Initialize	25 ms	25 ms
		Update	15 ms	7 ms
d.	No drag	Initialize	300 ms	300 ms
		Update	17 ms	8 ms

Test Case 2: High eccentricity (0.72729) orbit

Initial conditions: A = 24407.29 km   e = .72729   i = 28.6°
                     W = 0°   Ω = 20°   M = 0°

			Stop on time[1]	Stop on revs[1]
a.	8 x 8	Initialize	750 ms	750 ms
		Update	30 ms	30 ms
b.	8 x 0	Initialize	18 ms	18 ms
		Update	30 ms	10 ms
c.	5 x 2	Initialize	30 ms	30 ms
		Update	25 ms	7 ms

---

[1]All times refer to the execution of a FORTRAN V program on a UNIVAC 1110-EXEC 8 system. The execution time will also depend on the computer environment at the time of execution.

APPENDIX E

GENERAL VARIABLE ABBREVIATIONS AND DEFINITIONS

PS elements:

Coordinates σ_1, σ_2, σ_3, σ_4

(Note: $\sigma_5 = \rho_1$, $\sigma_6 = \rho_2$, $\sigma_7 = \rho_3$, $\sigma_8 = \rho_4$)

Momenta ρ_1, ρ_2, ρ_3, ρ_4

Independent variable τ (true anomaly)

PS Hamiltonian:

$$F = \rho_1 - \frac{\mu}{2\rho_4} + \frac{r^2}{q} V$$

where $q = G - 1/2\ \Phi + \frac{\mu}{2\sqrt{2L}}$, and V is the perturbing potential.

DS elements:

ϕ = true anomaly

g = argument of perigee

h = argument of the ascending node

ℓ = time element

Φ = conjugate to ϕ, related to the two-body energy

G = total angular momentum

H = Z component of the angular momentum

L = total energy (two-body plus perturbing potential)

(Note: For a complete description of the relationship between the DS and the PS elements, see reference 5.)

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Cartesian coordinates:

$\vec{X}$ = (X_x, X_y, X_z) = position vector

$\vec{V}$ = (V_x, V_y, V_z) = velocity vector

r = magnitude of the position vector

t = physical time

Keplerian elements:

a = semimajor axis

e = eccentricity

i = inclination to the equator

ω = argument of pericenter

Ω = argument of the ascending node

M = mean anomaly

Planetary variables: (see appendix A for the numerical values used)

R_e = equatorial radius

μ = gravitational constant

General:

km	= kilometers	min	= minutes
nm	= nautical miles	rad	= radians
ft	= ft	deg	= degrees
m	= meters	t	= time
E.r.	= Earth radius	$\longrightarrow$	= denotes a vector as $\vec{X}$
sec	= seconds		
hr	= hours		

APPENDIX F

EQUATIONS OF THE ANALYTICAL THEORY

A complete first-order solution for the motion of a satellite perturbed by oblateness has been developed (ref. 5). A brief outline was given in reference 17 and is reproduced in this appendix.

The Hamiltonian for the J_2 perturbed case can be written as

$$F = \rho_1 - \frac{\mu}{\sqrt{2\rho_4}} + \epsilon F_1$$

where

$$F_1 = 1/r \left[\left(\frac{x_3}{r} \right)^2 - \frac{1}{3} \right]$$

and

$$\epsilon = 3/2 \ (J_2 \ \mu \ R_e^2)$$

R_e is the mean equatorial radius of the central body; μ is the gravitational constant of the central body, and J_2 is the J_2 oblateness coefficient.

The differential equations are solved by a method of Von-Zeipel. The elements undergo a canonical transformation through a determining function S_1 so that the short periodic terms are eliminated from the Hamiltonian. The equations of motion in the transformed system $\vec{\sigma}'$ may then be solved with an accuracy of order ϵ.

The solution algorithm can be divided into three steps:

a. Canonical transformation to eliminate the short periodic terms:

$$\sigma'_{k,0} = \sigma_{k,0} + \epsilon \ \frac{\partial S_1}{\partial \rho_{k,0}} \cdot (\sigma_0, \ \rho_0)$$

$$\rho'_{k,0} = \rho_{k,0} - \epsilon \ \frac{\partial S_1}{\partial \sigma_{k,0}} \ (\sigma_0, \ \rho_0)$$

$$k = 1,2,3,4$$

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b. The analytical integration of the transformed equations of motion:

$$\sigma_1' = \sigma_{1,0}' + A_1 \tau$$

$$\sigma_2' = \sigma_{2,0}' \cos(A_2\tau) - \rho_{2,0}' \sin(A_2\tau)$$

$$\sigma_3' = \sigma_{3,0}' \cos(A_3\tau) - \rho_{3,0}' \sin(A_3\tau)$$

$$\sigma_4' = \sigma_{4,0}' + A_4\tau$$

$$\rho_1' = \rho_{1,0}'$$

$$\rho_2' = \rho_{2,0}' \cos(A_2\tau) + \sigma_{2,0}' \sin(A_2\tau)$$

$$\rho_3' = \rho_{3,0}' \cos(A_3\tau) + \sigma'_{3,0}\sin(A_3\tau)$$

$$\rho_4' = \rho_{4,0}'$$

c. The back transformation:

$$\sigma_k = \sigma_k' - \epsilon \frac{\partial S_1}{\partial \rho_{k'}}(\sigma',\rho')$$

$$\rho_k = \rho_k' + \epsilon \frac{\partial S_1}{\partial \sigma_{k'}}(\sigma',\rho')$$

$$k = 1,2,3,4$$

If one defines

$$S_{1k} = \frac{\partial S_1}{\partial \sigma_k} \qquad \text{where} \qquad S_1 = -\frac{1}{G^2} w\, y$$

then

$$S_{1k} = \frac{-1}{G^2}\left(w_k\, y + w\, y_k - \frac{2\, w\, y\, G_k}{G} \right)$$

where

$$w = \frac{Q}{2pq}$$

$$w_k = \frac{1}{2p^2q^2} \left[pq\, Q_k - Q\, (p_k q + q p_k) \right]$$

$$y = \sum_{\ell=1}^{3} (\delta_\ell\, \eta_\ell + \gamma_\ell\, \xi_\ell)$$

$$y_1 = \sum_{\ell=1}^{3} \ell\, (\delta_\ell\, \dot{\xi}_\ell - \gamma_\ell\, \eta_\ell) + \delta_{\ell 1}\eta_\ell + \gamma_\ell\xi_\ell$$

$$y_k = \sum_{\ell=1}^{3} (\delta_{\ell k}\eta_\ell + \gamma_{\ell k}\xi_\ell) \quad k = 2,3,\ldots,8$$

$$G = \sigma_5 - \frac{1}{2}\, (\sigma_2^{\ 2} + \sigma_6^{\ 2})$$

$$G_k = 0 \quad \text{for } k = 1,3,4,7,8$$

$$G_2 = -\sigma_2$$

$$G_5 = 1$$

$$G_6 = -\sigma_6$$

Here p, p_k, q, q_k, Q, Q_k, δ_ℓ, η_ℓ, γ_ℓ, ξ_ℓ and $\delta_{\ell k}$, $\gamma_{\ell k}$ are displayed

$$p = \frac{1}{\mu} \left[-\left(\frac{1}{2}\, \sigma_2^{\ 2} + \sigma_6^{\ 2} \right) + \frac{\mu}{\sqrt{2\sigma_8}} \right]^2$$

$$p_2 = -2\frac{\sqrt{\mu p}}{\mu}\, \sigma_2$$

$$p_6 = -2\frac{\sqrt{\mu p}}{\mu}\, \sigma_6$$

$$p_8 = -2\frac{\sqrt{\mu p}}{(2\sigma_8)^{3/2}}$$

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$$p_k = 0 \quad \text{for } k = 1,3,4,5,7$$

$$q = -\frac{1}{2}(\sigma_6{}^2 + \sigma_2{}^2 - \sigma_5) + \frac{\mu}{2\sqrt{2\sigma_8}}$$

$$q_2 = -\sigma_2$$

$$q_5 = \frac{1}{2}$$

$$q_8 = -\frac{\mu}{2} \frac{1}{(2\sigma_8)^{3/2}}$$

$$q_k = 0 \quad \text{for } k = 1,3,4,6,7$$

$$Q = \left\{ \frac{\sigma_8}{\mu^2} \left[\frac{2\mu}{2\sigma_8} - \frac{1}{2}(\sigma_2{}^2 + \sigma_6{}^2) \right] \right\}^{1/2}$$

$$Q_2 = -\frac{\sigma_8 \sigma_2}{2Q\mu^2}$$

$$Q_6 = -\frac{\sigma_8 \sigma_6}{2Q\mu^2}$$

$$Q_8 = \frac{\sqrt{\mu p}}{2Q\mu^2}$$

$$Q_k = 0 \quad \text{for } k = 1,3,4,5,7$$

$$\delta_1 = \frac{B}{3}\sigma_6 - \frac{1}{2}(\sigma_6 c - \sigma_2 s)$$

$$\delta_{12} = \frac{s}{2} + \frac{\sigma_6}{3}B_2 - \frac{1}{2}(\sigma_6 c_2 - \sigma_2 s_2)$$

$$\delta_{16} = \frac{B}{3} - \frac{c}{2} + \frac{\sigma_6}{3}B_6 - \frac{1}{2}(\sigma_6 c_6 - \sigma_2 s_6)$$

$$\delta_{1k} = \frac{\sigma_6}{3}B_k - \frac{1}{2}(\sigma_6 c_k - \sigma_2 s_k) \quad \text{for } k = 1,3,4,5,7,8$$

$$\gamma_1 = \frac{B}{3}\,\sigma_2 + \frac{1}{2}\,(\sigma_6 s + \sigma_2 c)$$

$$\gamma_{12} = \left(\frac{B}{3} + \frac{C}{2}\right) + \frac{\sigma_2}{3}\,B_2 + \frac{1}{2}\,(\sigma_6 s_2 + \sigma_2 c_2)$$

$$\gamma_{16} = \frac{s}{2} + \frac{\sigma_2}{3}\,B_6 + \frac{1}{2}\,(\sigma_6 s_6 + \sigma_2 c_6)$$

$$\gamma_{1k} = \frac{\sigma_2}{3}\,B_k + \frac{1}{2}\,(\sigma_6 s_k + \sigma_2 c_k) \quad \text{for} \quad k = 1,3,4,5,7,8$$

$$\delta_2 = -\frac{c}{2Q}$$

$$\delta_{2k} = \frac{1}{2Q}\left(\frac{c}{Q}\,Q_k - c_k\right) \text{for} \quad k = 1,2,3,\ldots,8$$

$$\gamma_2 = \frac{s}{2Q}$$

$$\gamma_{2k} = -\frac{1}{2Q}\left(\frac{s}{Q}\,Q_k - s_k\right) \quad \text{for} \quad k = 1,2,3,\ldots,8$$

$$\delta_3 = -\frac{1}{6}\,(\sigma_2 s + \sigma_6 c)$$

$$\delta_{32} = -\frac{1}{6}\,(\sigma_2 s_2 + \sigma_6 c_2 + s)$$

$$\delta_{36} = -\frac{1}{6}\,(\sigma_2 s_6 + \sigma_6 c_6 + c)$$

$$\delta_{3k} = -\frac{1}{6}\,(\sigma_2 s_k + \sigma_6 c_k) \quad \text{for} \quad k = 1,3,4,5,7,8$$

$$\gamma_3 = \frac{1}{6}\,(\sigma_6 s - \sigma_2 c)$$

$$\gamma_{32} = \frac{1}{6} \left(\sigma_6 s_2 - \sigma_2 c_2 - c \right)$$

$$\gamma_{36} = \frac{1}{6} \left(\sigma_6 s_6 - \sigma_2 c_6 + s \right)$$

$$\gamma_{3k} = \frac{1}{6} \left(\sigma_6 s_k - \sigma_2 c_k \right) \quad \text{for} \quad k = 1,3,4,5,7,8$$

$$\eta_\ell = \sin \ell \sigma_1$$

$$\xi_\ell = \cos \ell \sigma_1 \quad \text{for} \quad \ell = 1,2,3$$

Here c, s, c_k, s_k, B, B_k, H and H_k are displayed

$$c = (G + H) \left(\frac{\sigma_7^2 - \sigma_3^2}{2} \right)$$

$$c_3 = \frac{H_3 c}{(G + H)} - (G + H) \sigma_3$$

$$c_7 = \frac{H_7 c}{(G + H)} + (G + H) \sigma_7$$

$$c_k = \frac{G_k + H_k}{(G + H)} c \quad \text{for} \quad k = 1,2,4,5,6,8$$

$$s = -(G + H) \sigma_3 \sigma_7$$

$$s_3 = \frac{H_3 s}{(G + H)} (G + H) \sigma_7$$

$$s_7 = \frac{H_7 s}{(G + H)} (G + H) \sigma_3$$

$$s_k = \frac{(G_k + H_k)}{(G + H)} s \quad \text{for} \quad k = 1,2,4,5,6,8$$

$$B = G^2 - 3H^2$$

$$B_k = 2(GG_k - 3HH_k) \quad \text{for} \quad k = 1,2,3,\ldots,8$$

$$H \equiv G - \frac{1}{2}(\sigma_3{}^2 + \sigma_7{}^2)$$

$$H_3 = -\sigma_3$$

$$H_7 = -\sigma_7$$

$$H_k = G_k \quad \text{for} \quad k = 1,2,4,5,6,8$$

Abbreviations used in the integration of the primed system

$$A_4 = \frac{\varepsilon}{2}f_4(b - \frac{2}{3}) + \frac{\mu}{(2\sigma_8)^{3/2}}$$

$$A_3 = \frac{\varepsilon}{2}fb_3$$

$$A_2 = \frac{\varepsilon}{2}\left[f_2\left(b - \frac{2}{3}\right) + fb_2\right] + A_3$$

$$A_1 = 1 + \frac{\varepsilon}{2}f_1\left(b - \frac{2}{3}\right) + A_2$$

$$f = \frac{1}{pq}$$

$$f_1 = \frac{f^2}{\mu}\left(\frac{1}{2}\mu p + 2q\sqrt{\mu p}\right)$$

$$f_2 = -\frac{f^2}{\mu}(\mu p + 2q\sqrt{\mu p})$$

$$f_4 = \frac{f^2}{(2\rho_4)^{3/2}}\left(\frac{1}{2}\mu p + 2q\sqrt{\mu p}\right)$$

$$b = 1 - \left(\frac{H}{G}\right)^2$$

$$b_2 = \frac{2}{G}\left(\frac{H}{G}\right)^2$$

$$b_3 = -\frac{2}{G}\left(\frac{H}{G}\right)$$

APPENDIX G

STANDARD FORTRAN VARIABLES USED IN ASOP

FORTRAN variable	Program location	Description
AEI(6)	OUTPUT	Character array to accompany Keplerian element output
ANG(3)	OUTPUT	Character array to accompany any angular output
B(3,3)	AEIXYZ	Keplerian elements to Cartesian coordinates transformation matrix
BS	/S1STAV/	$1 - H/G$
BY3	/CONSTW/	$1/3$
BY6	/CONSTW/	$1/6$
C(8)	PSANS	$\partial c/\partial \sigma_k$, $\partial c/\partial \rho_k$, $k = 1,2,3,4$
CHECK	OUTPUT	Energy check value
CINC	AEIXYZ	Cosine of the orbital inclination with respect to the Earth's equator ($\cos i$)
CN	/CONSTW/	± 1 depending on value of NN
CNODE	AEIXYZ	Cosine of the argument of the ascending node ($\cos \Omega$)
COMEGA	AEIXYZ	Cosine of the argument of pericenter ($\cos \omega$)
COSEA	AEIXYZ	Cosine of the eccentric anomaly ($\cos E$)
COSFC2	PSANS	$\cos (A_2\tau)$ (see appendix F)
COSFC3	PSANS	$\cos (A_3\tau)$ (see appendix F)
CS	PSANS	$(1/2) (G + H) (\rho_3^2 + \sigma_3^2) = $ small 'c'
DAYS(7)	CONST	Storage array of possible values of DAY
DAYS	OUTPUT	Print value (DAYS = TIME/DAY)
DAYS	TIMEPS	Total days elapsed

FORTRAN variable	Program location	Description
DELTA1	PSANS	δ_1
DELTA2	PSANS	δ_2
DELTA3	PSANS	δ_3
DEL1(8)	PSANS	$\partial\delta_1/\partial\sigma_k$, $\partial\delta_1/\partial\rho_k$ $k = 1,2,3,4$
DEL2(8)	PSANS	$\partial\delta_2/\partial\sigma_k$, $\partial\delta_2/\partial\rho_k$ $k = 1,2,3,4$
DEL3(8)	PSANS	$\partial\delta_3/\partial\sigma_k$, $\partial\delta_3/\partial\rho_k$ $k = 1,2,3,4$
DSB(4)	PSANS	$\partial b/\partial\beta_k$ $k = 1,2,3,4$
DSF(4)	PSANS	$\partial f/\partial\beta_k$ $k = 1,2,3,4$
DST(7)	OUTPUT	Character array to accompany any distance output
EA	$\left\{\begin{array}{c}\text{AEIXYZ}\\\text{XYZAEI}\end{array}\right\}$	The eccentric anomaly of the satellite computed from Kepler's equation (E) (rad)
EAO	AEIXYZ	Old value of EA; used when iterating to solve Kepler's equation (rad)
ECOSE	XYZAEI	E cos e
ECOSPH	$\left\{\begin{array}{c}\text{PSTOX}\\\text{XTOPS}\end{array}\right\}$	E cos ϕ
EL(6)	INPUT	Initial conditions of the satellite given in Keplerian elements or Cartesian coordinates; on output, it will contain the Keplerian elements. EL(1) X or a (2) Y or e (3) Z or i (4) X or ω (5) Y or Ω (6) Z or M
EMINPH	$\left\{\begin{array}{c}\text{PSTOX}\\\text{XTOPS}\end{array}\right\}$	E $-$ Φ
EROOT	/PSANS3/	$\sqrt{2\rho_4\ p/\mu}$
EROOT	XTOPS	$\sqrt{1 - 2Q(\Phi - G)}$

FORTRAN variable	Program location	Description
ESINE	XYZAEI	$E \sin e$
ESINPH	$\{ \begin{matrix} PSTOX \\ XTOPS \end{matrix} \}$	$E \sin \phi$
ETA2	PSANS	$\sin 2\sigma_1$
ETA3	PSANS	$\sin 3\sigma_1$
FACTOR(4)	PSANS	Derivatives of the DS Hamiltonian and its combinations (A_1, A_2, A_3, A_4) (see appendix F)
FN(19,19)	POTEXP	Inclination function
FS	/S1STAV/	f
FSSQ	PSANS	f^2
GAMMA1	PSANS	γ_1
GAMMA2	PSANS	γ_2
GAMMA3	PSANS	γ_3
GAM1(8)	PSANS	$\partial\gamma_1/\partial\sigma_k$, $\partial\gamma_1/\partial\rho_k$ $k = 1,2,3,4$
GAM2(8)	PSANS	$\partial\gamma_2/\partial\sigma_k$, $\partial\gamma_2/\partial\rho_k$ $k = 1,2,3,4$
GAM3(8)	PSANS	$\partial\gamma_3/\partial\sigma_k$, $\partial\gamma_3/\partial\rho_k$ $k = 1,2,3,4$
GC(8)	/S1STAD/	$\partial G/\partial\sigma_k$, $\partial G/\partial\rho_k$ $k = 1,2,3,4$
GCAP(17)	POTEXP	Eccentricity function
GCIN	PSTOX	G^{-1}
GCSQ	XTOPS	G^2
GIN	/S1STAV/	G^{-1}
GINSQ	/S1STAV/	G^{-2}
GM3H	PSANS	$G - 3H$
GPH	/S1STAV/	$G + H$
GSQ	PSANS	G^2

FORTRAN variable	Program location	Description
G1	XTOPS	$YV_z - ZV_y = G_x$
G1SQ	XTOPS	G_x^2
G2	XTOPS	$ZV_x - XV_z = G_y$
G2SQ	XTOPS	G_y^2
G3	XTOPS	$XV_y - YV_x = G_z$
H	XYZAEI	Total angular momentum
HC(8)	/S1STAD/	$\partial H/\partial \sigma_k$, $\partial H/\partial \rho_k$ k = 1,2,3,4
HMS(4)	OUTPUT	Character array to accompany any time output
HOG	/SISTAV/	H/G
HSQ	PSANS	H^2
IA(3)	OUTPUT	Character array of blanks and asterisks
IEL	INPUT	Flag to determine if input values of EL are given as Keplerian elements or Cartesian coordinates = 1 Keplerian = 2 Cartesian
IERR	TIMEPS	Error counter
IFORM	OUTPUT	Flag to determine if initial or final condition messages is to be printed = 1 initial condition message = 2 no message (intermediate print) = 3 final condition message
IMARK	PSANS	Flag determining if one or two passes have been made = 1 1st pass = 2 2nd pass

FORTRAN variable	Program location	Description
ITER	TIMEPS	Total number of iterations allowed
IXP	OUTPUT	Pointer to the IA array
		IFORM $\neq$ 3 $\rightarrow$ IXP = IP
		IFORM = 3 $\rightarrow$ IXP = ISTOP
L	PSANS $\Big\}$	$L = \rho_4 = \sigma_8$
LC	XTOPS	
LS	PSANS	$\ell = \sigma_4$
NEWX	$\Big\{$ MAIN ASOP $\Big\}$	Flag to determine if ASOP program is to be initialized
		= 0 no
		= 1 yes
NN	PSANS	Flag to determine if initializing or computing
		= 0 initializing
		= 1 computing
P	XYZAEI	$1 - e^2$
P(8)	/S1STAD/	$\partial p / \partial \sigma_k$, $\partial p / \partial \rho_k$ k = 1,2,3,4
PHI	PSANS $\Big\}$	$\Phi = \rho_1 = \sigma_5$
PHIC	XTOPS	
POT	$\Big\{$ GEOPOT PSTOX XTOPS $\Big\}$	Magnitude of Earth's gravitational potential

FORTRAN variable	Program location	Description
P5	$\left\{\begin{array}{l}\text{PSANS}\\\text{PSTOX}\end{array}\right\}$	1/2
Q(8)	/S1STAD/	$\partial q/\partial\sigma_k$, $\partial q/\partial\rho_k$ k = 1,2,3,4
QCIN	PSANS	Q^{-1}
QCSQ	PSANS	Q^2
QCV(8)	/S1STAD/	$\partial Q/\partial\sigma_k$, $\partial Q/\partial\rho_k$ k = 1,2,3,4
QS	/PSANS2/	$1/2\left(\dfrac{\mu}{2L} - G\right)$
RCAP	$\left\{\begin{array}{l}\text{PSTOX}\\\text{XTOPS}\end{array}\right\}$	R (see section 4.2)
RCAPDT	PSTOX XTOPS	$\partial R/\partial t$ (see section 4.2)
RCOSF	XYZAEI	r cos f; f = true anomaly, r = R
RCOSL	XYZAEI	r cos L; L = mean anomaly, r = R
RDOT	PSTOX	Magnitude of velocity vector
RES(7)	CONST	Storage array of possible values of RE
REVS	OUTPUT	Total number of revolutions predicted (REVS = TAU/2_π)
ROP	PSTOX	r/p (see section 4.2)
RRDOT	XYZAEI	$X \cdot V$
RSINF	XYZAEI	r sin f, f = true anomaly, r = R
RSINL	XYZAEI	r sin L, L = mean anomaly, r = R
RSQ	XTOPS	R^2
R2I	GEOPOT	$1/R^2$
S(8)	PSANS	$\partial s/\partial\sigma_k$, $\partial s/\partial\rho_k$ k = 1,2,3,4
SINC	AEIXYZ	Sine of the orbital inclination with respect to the Earth's equator (sin i)

FORTRAN variable	Program location	Description
SIGINI(8)	PSANS	Initial values of σ's and ρ's
SINEA	AEIXYZ	Sine of the eccentric anomaly (sin E)
SINFC2	PSANS	sin $(A_2 \tau)$ (see appendix F)
SINFC3	PSANS	sin $(A_3 \tau)$ (see appendix F)
SINWF	XYZAEI	sin $(\omega + f)$ ω = argument of perigee, f = true anomaly
SNODE	AEIXYZ	Sine of the argument of the ascending node (sin Ω)
SOMEGA	AEIXYZ	Sine of the argument of pericenter (sin ω)
SQTGHI	XTOPS	$-\sqrt{2/(G + H)}$
SS	PSANS	s
STOPDT	MAIN	Value at which next intermediate printout is desired (needed only if IPRINT $\geq$ 1)
SUM2	/PSANS2/	$1/2\ (\sigma_2{}^2 + \rho_2{}^2) = \Phi - G$
SUM3	/PSANS2/	$1/2\ (\sigma_3{}^2 + \rho_3{}^2) = 2\ (G - H)$
S1(4)	PSANS	Derivatives of the generating function S_1
TFIN	TIMEPS	Final time desired for stopping the iteration
TWO3	/CONSTW/	2/3
VEL(7)	OUTPUT	Character array to accompany any velocity output
V1(3)	AEIXYZ	Velocity vector with respect to the orbital plane $(V_x', V_y', V_z' = 0)$
VSQ	XYZAEI	Magnitude of the velocity vector, squared (V^2)
W(8)	PSANS	$\partial w/\partial \sigma_k, \partial w/\partial \rho_k$ k = 1,2,3,4
WS	PSANS	w

| FORTRAN variable	Program location	Description
WX WY WZ	XYZAEI	Components of the total angular momentum (WX, WY, WZ) (distance^2/time)
XMUS(7)	CONST	Storage array of possible values of XMU
XIN(8)	ASOP	Identical to X but allows ASOP subroutine to be removed from the stand alone program
XYZ(6)	OUTPUT	Character array to accompany output of the Cartesian state vector
X1(3)	AEIXYZ	Position vector of the satellite with respect to the orbital plane (X', Y', Z' = 0)
X3ROOT	/PSANS3/	$\left(\sqrt{4G - \sigma_3^2 - \rho_3^2}\right)$/G (see section 4.2)
Y	PSANS	$\partial y/\partial \sigma_k, \; \partial y/\partial \rho_k \quad k = 1,2,3,4$
YS	PSANS	y
ZCAP1	XTOPS	z_1
ZCAP2	XTOPS	z_2
ZET2	PSANS	$\cos 2\sigma_1$
ZET3	PSANS	$\cos 3\sigma_1$

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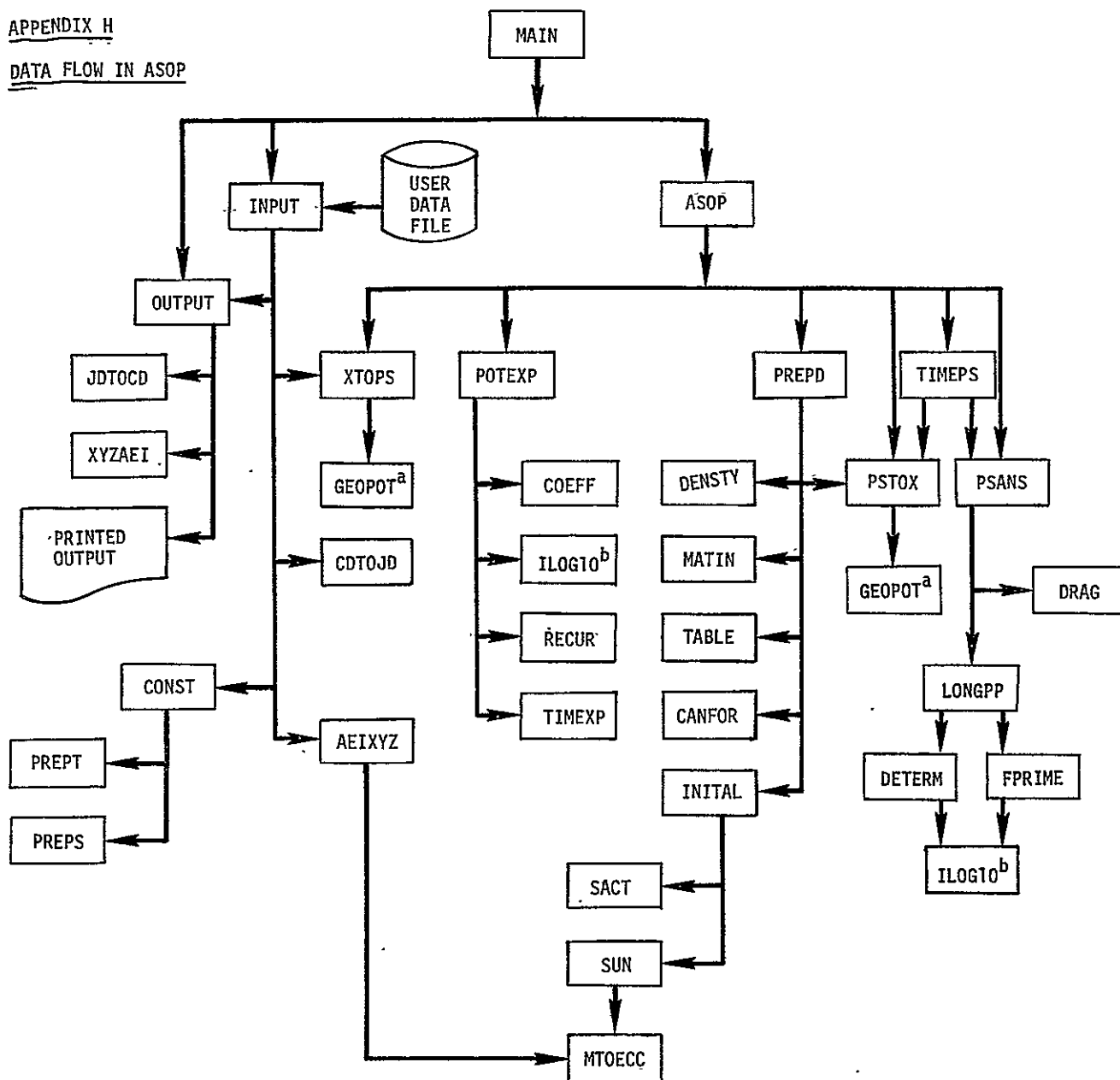
Figure 41.- Data flow in ASOP-general subroutine linkage.

[a]GEOPOT subroutines are the same

[b]ILOG10 subroutines are the same
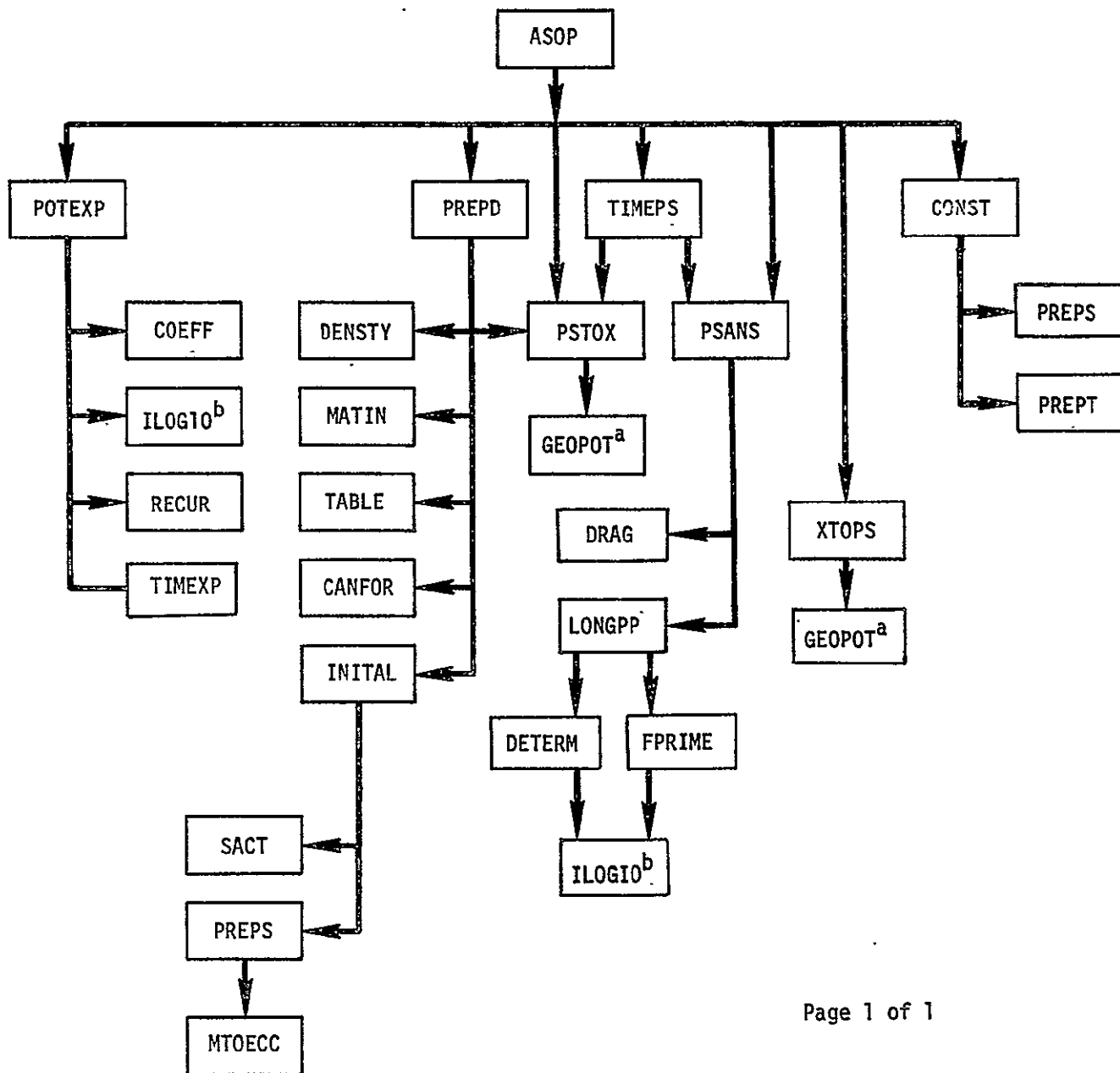
Figure 42.- General subroutine linkage in
removable ASOP subroutine package.

[a]GEOPOT subroutines are the same

[b]ILOG10 subroutines are the same

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